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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

FUNCTIONAL DESIGN OF FLOOD CONTROL RESERVOIRS

By C. J. Posey,[1] Jun. Am. Soc. C. E., and Fu-Te I,[2] Esq.

Synopsis

The functional design of a flood control reservoir has as its objective the determination of the relation between the storage space that must be provided and the corresponding reduction of the flood peak. The method of incorporating this relation into the study of the economic balance of a flood control system has been explained in detail by Sherman M. Woodward,[3] M. Am. Soc. C. E. Mr. Woodward's "five-sixths rule," which heretofore has provided the only direct method for the functional design computations, is limited in its application to reservoirs of such proportions that the maximum outflow is small compared with the average inflow during the flood, and to reservoirs with orifice-type outlets and a certain type of depth-capacity relationship. In this paper the writers have generalized Mr. Woodward's method and have extended it to apply to reservoirs with either orifice- or weir-type outlets, in valleys of a wide range of morphological configuration. Although the relationships derived can be used in the design of multiple-purpose reservoirs, the present discussion is restricted to reservoirs for flood control only.

Introduction

A necessary preliminary step is the estimation of the maximum flood runoff that may occur at the given site. This is usually based upon a statistical study of meteorological and hydrological data, and includes a determination of the shape of the flood hydrograph, as well as its volume. Methods of making these studies have been the subject of extended discussion in recent years,[4]

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February 15, 1940.**

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[2] Graduate Student, Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

[3] "Hydraulics of the Miami Flood Control Project," by Sherman M. Woodward, Technical Reports of the Miami Conservancy District, State of Ohio (1920), Part VII, Chapter XII, p. 286.

[4] Technical Reports of the Miami Conservancy District, Part V; also "Analysis of Run-Off Characteristics," by Otto H. Meyer, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., November, 1938, p. 1769.

and further treatment is not necessary in this paper. Suffice to it say that the flood hydrograph used for the design is made somewhat greater than the maximum probable flood to provide a margin of safety. No matter how carefully this work is done, the design flood will have the kind of uncertainty that is attached to any prediction of future events. This sets a limit of accuracy beyond which it is useless to carry subsequent computations.

In order to determine the storage necessary to accomplish a desired reduction of peak flood flow, it is necessary to route the design flood through the reservoir—that is, to compute the moderating effect of the reservoir storage upon the flood hydrograph.

In general, flood-routing methods are of two types:

(1) Step methods, in which the flood inflow hydrograph is divided into steps or time intervals of such short duration that during each one the inflow and outflow rates may be assumed to be constant. The actual depth-capacity data for the reservoir site are used and the discharge-depth relation must be known. An irregular inflow hydrograph may be approximated, as closely as desired, by using a large number of steps.

(2) Integration methods, in which the given flood hydrograph is replaced by an equivalent flood of uniform intensity. The depth-capacity data are represented by an empirical formula. It is necessary to know only the exponent and approximate zero-depth of the discharge-depth relation.

The first step methods to be used required a trial-and-error solution of each step. Later, graphical methods were developed which eliminated this cut-and-try process.[5] Results of the step computations include the outflow and stage at the end of each step, so that step methods are well adapted for operation studies. In application to functional design, however, step methods are indirect. It is necessary first to assume the size of the outlet. The corresponding maximum storage and maximum outflow are determined by means of routing computations. Another size of outlet is then assumed, and the process is repeated until the desired part of the curve representing the relation between storage and percentage reduction in flood peak has been obtained. On the other hand, integration methods give the storage required for a given maximum outflow directly, with considerable saving of time and labor.[6]

Depth-Capacity Exponent.—The relation between depths and capacities in the reservoir may be represented by an empirical expression of the form

$$s = C\,h^m \quad\quad (1)$$

in which: s is the storage capacity of the reservoir at a depth h; h is the depth of water above outlet; C is a constant; and m is the "depth-capacity" exponent. For the determination of m, logarithmic plotting is used. The depths above outlet location (zero depth of the orifice type of outlet, or the crest of a weir type of outlet) are plotted as ordinates and the corresponding capacities as abscissas. A straight line best fitting the plotted points is drawn. The exponent m is the reciprocal of the slope of the line, the slope being measured directly with a scale having uniformly spaced divisions, disregarding the logarithmic scale of the plot. The constant C is not needed in the analysis.

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[5] "Slide Rule for Routing Floods through Storage Reservoirs or Lakes," by C. J. Posey, *Engineering News-Record*, April 25, 1935, Vol. 114, p. 580.

[6] "Hydraulics of the Miami Conservancy District," by Sherman M. Woodward, Technical Reports of the Miami Conservancy District, Part VII, Chapter VII.

A similar procedure may be followed with the depth-area data, and the depth-capacity exponent found by integration of the empirical relation so determined. It is better, however, to determine the depth-capacity exponent directly, because integration of the empirical relation derived by logarithmic plotting is more likely to introduce error than numerical summation of the original survey data.

In studying the depth-capacity curves for more than 250 reservoir sites, m was found to vary from 1.03 to 4.74. The following classification, given by R. A. Sutherland, Assoc. M. Am. Soc. C. E., was found to be justified.[7]

Classification	Values of m
Lake type	1.0 to 1.5
Flood-plain and foothill type	1.5 to 2.5
Hill type	2.5 to 3.5
Gorge type	3.5 to 4.5

In almost all cases, the points lay very nearly along a straight line on the logarithmic plot. The use of the exponential formula to obtain capacities at higher levels than those for which survey data are available is inadvisable. For some sites, the logarithmic plot will follow a straight line over a considerable depth, and then deviate sharply to one side. For others, the points will lie on a curve. Since volumes near the bottom of the reservoir are relatively insignificant, it is important that the best straight line should fit the data at the higher levels most closely. If the straight line is a close approximation to the data at higher levels, discrepancies at the lower levels can be ignored, with inappreciable error in the results. Some judgment is required in deciding whether the degree of approximation is satisfactory. No rigid rule can be formulated. In doubtful cases a check should be made by the step method.

Equivalent Uniform Flood.—The equivalent uniform flood corresponding to a given irregular flood hydrograph is a flood of uniform intensity, having the same total inflow as the given flood, which will produce the same total amount of storage with the same maximum rate of outflow from the reservoir. It is possible to determine a uniform flood exactly equivalent to a given irregular flood by detailed routing computations. It has been found, however, that even for hydrographs differing considerably from rectangular, it is not difficult to select, arbitrarily, an equivalent uniform flood that will give results sufficiently accurate for design purposes. For example, consider the triangular flood shown in Fig. 1. This flood was routed by the step method through a reservoir ($m = 2.5$) with the orifice type of outlet. The maximum outflow reached was 5,160 acre-ft per hr and the maximum storage, 117,100 acre-ft.

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[7] "Some Aspects of Water Conservation," by R. A. Sutherland, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 164.

If the duration T of an equivalent uniform flood is assumed to be 40 hr, the intensity I should be equal to the total volume of the flood divided by 40, or 7,320 acre-ft per hr. Routing this uniform flood through the reservoir by the step method the maximum rate of outflow reached was found to be 5,140 acre-ft per hr, which is very close to the value obtained for the triangular flood.

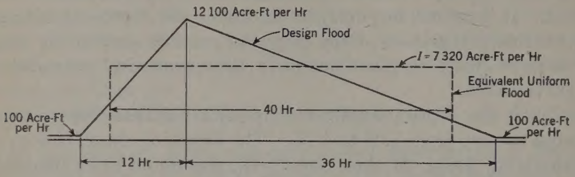

FIG. 1.—TRIANGULAR FLOOD AND EQUIVALENT UNIFORM FLOOD

The error in the maximum outflow rate and the maximum storage, corresponding to different assumptions for the duration of the uniform flood, is given in Table 1. It is seen that any estimate for the duration of the equivalent uniform flood between the values of 38 hr and 42 hr gives satisfactory accuracy, in this case. Similar results from other comparisons made by the writers confirm the belief that arbitrary selection of equivalent uniform floods is not difficult. It should be remembered that there is considerable uncertainty in the shape and volume of the "design" hydrograph.

TABLE 1.—ERRORS IN THE MAXIMUM OUTFLOW RATE AND MAXIMUM STORAGE

Duration assumption, T, in hours (1)	Inflow, I, in acre-feet per hour (2)	Maximum outflow rate, O, in acre-feet per hour (3)	Percentage error in maximum outflow rate (4)	Maximum storage, S, in acre-feet (5)	Percentage error in maximum storage (6)
38	7,700	5,195	+0.68	121,000	+3.3
40	7,320	5,140	−0.39	115,000	−1.8
42	6,970	5,080	−1.55	108,000	−7.8

Outflow Ratio.—The outflow ratio, x, is the ratio of the maximum outflow from the reservoir to the uniform inflow of the rectangular flood. It is the same as the "protection ratio" defined by Mr. Woodward.[6] The writers recommend that the name "protection ratio" be abandoned as misleading. The higher the value of this ratio, the greater the maximum outflow and the less the protection.

THE DETENTION RATIO

The detention ratio is the ratio of the maximum reservoir storage reached during the flood to the total volume of the uniform flood. By definition, then,

$$d = \frac{S}{I\,T} \qquad (2)$$

in which d is the detention ratio, S is the maximum storage, and $I\ T$ is the total volume of the uniform flood. The detention ratio is a function of the depth-capacity exponent, n, the outflow ratio, x, and the exponent governing the discharge of the outlets. The writers have computed values of d over the practical range of the outflow ratio and depth-capacity exponent, for reservoirs with orifice-type outlets (Table 2(a)) and for reservoirs with weir-type outlets (Table 2(b)). The values of d were computed with the aid of the infinite series developed in the Appendix. They can be checked (indirectly) by any of the standard step methods.

TABLE 2.—VALUES OF THE DETENTION RATIO FOR RESERVOIRS

Values of m (1)	$x = 0.1$ (2)	$x = 0.2$ (3)	$x = 0.3$ (4)	$x = 0.4$ (5)	$x = 0.5$ (6)	$x = 0.6$ (7)	$x = 0.7$ (8)	$x = 0.8$ (9)
(a) ORIFICE-TYPE OUTLETS								
1.0	0.933	0.864	0.794	0.722	0.647	0.570	0.489	0.399
1.5	0.925	0.848	0.771	0.692	0.611	0.528	0.441	0.348
2.0	0.920	0.839	0.757	0.674	0.590	0.504	0.415	0.322
2.5	0.916	0.832	0.748	0.662	0.575	0.488	0.397	0.303
3.0	0.914	0.828	0.741	0.654	0.565	0.476	0.384	0.290
3.5	0.912	0.824	0.736	0.647	0.557	0.467	0.374	0.279
4.0	0.911	0.822	0.732	0.642	0.551	0.460	0.366	0.271
5.0	0.909	0.818	0.726	0.635	0.542	0.450	0.355	0.258
6.0	0.908	0.815	0.722	0.630	0.536	0.442	0.347	0.250
(b) WEIR-TYPE OUTLETS								
1.0	0.959	0.916	0.870	0.822	0.769	0.710	0.645	0.567
1.2	0.955	0.907	0.858	0.805	0.748	0.686	0.618	0.536
1.4	0.951	0.900	0.846	0.790	0.730	0.665	0.593	0.509
1.6	0.948	0.893	0.836	0.777	0.714	0.647	0.573	0.486
1.8	0.945	0.887	0.828	0.766	0.700	0.632	0.555	0.467
2.0	0.942	0.882	0.821	0.756	0.689	0.617	0.539	0.451
2.2	0.940	0.878	0.814	0.748	0.679	0.605	0.525	0.437
2.4	0.938	0.874	0.808	0.740	0.669	0.594	0.514	0.425
2.6	0.936	0.870	0.803	0.734	0.661	0.585	0.504	0.414
2.8	0.934	0.867	0.798	0.728	0.654	0.577	0.495	0.404
3.0	0.933	0.864	0.794	0.722	0.647	0.569	0.486	0.395
3.5	0.930	0.858	0.785	0.710	0.633	0.553	0.468	0.377
4.0	0.927	0.853	0.777	0.700	0.621	0.539	0.453	0.362
4.5	0.925	0.848	0.771	0.692	0.611	0.528	0.441	0.349
5.0	0.923	0.844	0.766	0.685	0.603	0.519	0.431	0.338
5.5	0.921	0.841	0.761	0.679	0.596	0.511	0.422	0.330
6.0	0.920	0.839	0.757	0.674	0.590	0.504	0.415	0.322

It is worth noting that for manually controlled outlets operated at 100% efficiency,

$$d = 1 - x \quad \ldots\ldots (3)$$

ILLUSTRATIVE EXAMPLES

The significance and utility of the detention ratio in the functional design of flood control reservoirs will be illustrated by two examples.

Example 1.—Find the relation between the maximum outflow and the storage required for controlling a flood of 50,000 acre-ft in 24 hr by an orifice-outlet reservoir at reservoir site A, Fig. 2. The depth-capacity exponent for this site is 2.25. The average inflow is $I = \frac{50{,}000}{24} = 2{,}080$ acre-ft per hr. The computations are shown in Table 3, the column headings of which are self-explanatory. The curve showing the relation between the maximum outflow and the storage required can be plotted from the data in Columns (2) and (4).

TABLE 3.—COMPUTATIONS, EXAMPLE 1

Outflow ratio, x (1)	Maximum outflow, $O = x\,I$, in acre-feet per hour (2)	Detention ratio, d, from Table 2(a) (3)	Storage required, in acre-feet (S = 50,000 d) (4)
0.4	835	0.668	33,400
0.5	1,040	0.582	29,100
0.6	1,250	0.496	24,800
0.7	1,460	0.406	20,300
0.8	1,670	0.312	15,600

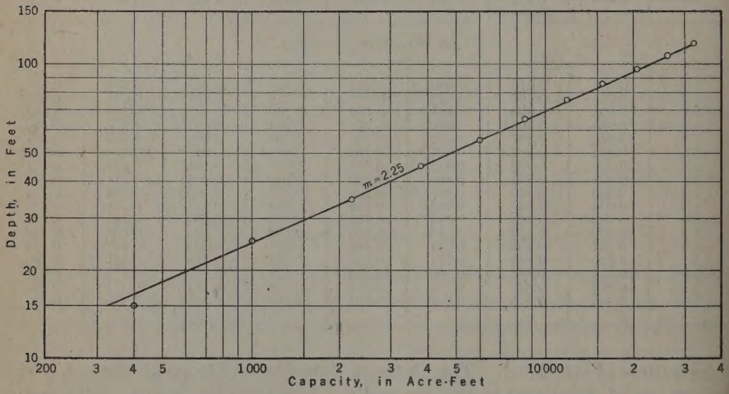

FIG. 2.—DEPTH-CAPACITY CURVE FOR RESERVOIR SITE "A"

Example 2.—Assume that, for the same example, political or other non-engineering reasons dictate that the maximum storage available shall be 25,000 acre-ft. What is the maximum flood reduction which may be obtained by the construction of a reservoir with an orifice type of outlet? By Equation (2), $d = \frac{25{,}000}{50{,}000} = 0.50$. From Table 2($a$)—for $m = 2.25$, and $d = 0.50$—$x = 0.605$. Hence, the maximum outflow $= x\,I = 0.605 \times 2{,}080 = 1{,}260$ acre-ft per hr. If the weir type of outlet is used instead of the orifice type, from Table 2(b)—for $m = 2.25$, and $d = 0.50$—$x = 0.725$. Hence, the maximum outflow $= 0.725 \times 2{,}080 = 1{,}510$ acre-ft per hr.

Manually-Controlled Outlets.—There has been much argument as to the relative merits of manually-controlled and automatic (fixed) outlet structures for flood control reservoirs. Comparisons based upon items as difficult to evaluate as fallibility should be postponed until the relative economy has been investigated. Tabulated values of the detention ratio afford an easy approach to such study. For example, assume that reservoir site A of Example 1 is to be used to reduce the maximum outflow from a flood of 50,000 acre-ft in 24 hr to 1,000 acre-ft per hr, and that the average cost of storage near the top of the reservoir is \$30 per acre-ft. By interpolation in Table 3 the storage required, if orifice-type outlets are used, is found to be 29,900 acre-ft. With manually-controlled outlets, an efficiency of 100% could presumably be reached, in which case 25,900 acre-ft of storage would be required. The saving in storage is 4,000 acre-ft, representing \$120,000. If manually-controlled outlets are to be economical, this \$120,000 must exceed the capitalized cost of the maintenance and operation of the mechanical gates, plus or minus any difference in the original cost of the outlets.

ACKNOWLEDGMENTS

The writers are indebted to Mr. Woodward, pioneer in the development of methods for flood routing, whose derivation of an expression similar to Equation (19) of the Appendix provided the clue that led to the development of the concept of the detention ratio, and to its evaluation. They are also indebted to E. W. Lane, M. Am. Soc. C. E., and Joseph W. Howe, Assoc. M. Am. Soc. C. E., for helpful suggestions and criticisms. This paper summarizes the principal findings of a thesis submitted by Mr. I to the Graduate College of the State University of Iowa, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

APPENDIX

Evaluation of the Detention Ratio.—The inflow into a retarding basin, during a small interval of time, is equal to the sum of the outflow and storage during that interval of time, evaporation and percolation losses being neglected. Thus,

$$i\,dt = o\,dt + ds \quad \text{.......................} (4)$$

in which: i = average inflow rate during the time dt; o = average outflow rate during the time dt; dt = a small increment of time; and ds = increment of storage capacity during the time dt. The outflow from a reservoir can be expressed in terms of the depth h:

$$o = B\,h^n \quad \text{.........................} (5)$$

in which n is 0.5 for the orifice type of outlet, and 1.5 for the weir type. The storage capacity can be approximated by a monomial exponential function of the depth:

$$s = C\,h^m \quad \text{......(1)}$$

so that

$$ds = C\,m\,h^{m-1}\,dh \quad \text{......(6)}$$

Substituting these relations into Equation (4),

$$i\,dt = B\,h^n\,dt + C\,m\,h^{m-1}\,dh \quad \text{......(7)}$$

For a constant inflow, I, during the entire flood period, Equation (7) becomes

$$I\,dt = B\,h^n\,dt + C\,m\,h^{m-1}\,dh \quad \text{......(8)}$$

Transposing and collecting like terms,

$$dt = \frac{C\,m}{I}\left(\frac{h^{m-1}}{1 - \frac{B}{I}\,h^n}\right)dh \quad \text{......(9)}$$

Equation (9) can be transformed into an infinite series by direct division,

$$dt = \frac{C\,m}{I}\left[h^{m-1} + \left(\frac{B}{I}\right)h^{m+n-1} + \left(\frac{B}{I}\right)^2 h^{m+2n-1} + \left(\frac{B}{I}\right)^3 h^{m+3n-1} + \cdots\right]dh \quad \text{......(10)}$$

Integrating, term by term,

$$t = \frac{C\,m}{I}\left[\frac{h^m}{m} + \left(\frac{B}{I}\right)\frac{h^{m+n}}{m+n} + \left(\frac{B}{I}\right)^2\frac{h^{m+2n}}{m+2\,n} + \left(\frac{B}{I}\right)^3\frac{h^{m+3n}}{m+3\,n} + \cdots\right] \quad \text{......(11)}$$

Substituting $o = B\,h^n$ and $s = C\,h^m$

$$t = \frac{s\,m}{I}\left[\frac{1}{m} + \left(\frac{o}{I}\right)\frac{1}{m+n} + \left(\frac{o}{I}\right)^2\frac{1}{m+2\,n} + \left(\frac{o}{I}\right)^3\frac{1}{m+3\,n} + \cdots\right] \quad \text{..(12)}$$

At the end of the uniform flood, when the elapsed time has reached the value T, the storage of water in the reservoir will have attained its maximum value S, and the outflow rate its maximum value O. At this instant, Equation (12) becomes

$$T = \frac{S\,m}{I}\left[\frac{1}{m} + \left(\frac{O}{I}\right)\frac{1}{m+n} + \left(\frac{O}{I}\right)^2\frac{1}{m+2\,n} + \left(\frac{O}{I}\right)^3\frac{1}{m+3\,n} + \cdots\right] \quad \text{..(13)}$$

If the ratio of the maximum outflow to the uniform inflow is denoted by x, the "outflow ratio," the total inflow during the flood period will be

$$I\,T = S\,m\left(\frac{1}{m} + \frac{x}{m+n} + \frac{x^2}{m+2\,n} + \frac{x^3}{m+3\,n} + \cdots\right)\ldots\ldots(14)$$

The total outflow during the reservoir-filling period, T, is equal to the total inflow minus the maximum storage,

$$\int o\,dt = I\,T - S = S\,m\left(\frac{x}{m+n} + \frac{x^2}{m+2\,n} + \frac{x^3}{m+3\,n} + \cdots\right)\ldots(15)$$

The average outflow during the period T is

$$\frac{\int o\,dt}{T} = \frac{I\left(\frac{x}{m+n} + \frac{x^2}{m+2\,n} + \frac{x^3}{m+3\,n} + \cdots\right)}{\left(\frac{1}{m} + \frac{x}{m+n} + \frac{x^2}{m+2\,n} + \frac{x^3}{m+3\,n} + \cdots\right)}\ldots\ldots(16)$$

The ratio, r, of the average outflow to the maximum outflow is given by

$$r = \frac{\frac{\int o\,dt}{T}}{O} = \frac{\frac{\int o\,dt}{T}}{x\,I}\ldots\ldots(17)$$

so that

$$r = \frac{\left(\frac{1}{m+n} + \frac{x}{m+2\,n} + \frac{x^2}{m+3\,n} + \cdots\right)}{\frac{1}{m} + x\left(\frac{1}{m+n} + \frac{x}{m+2\,n} + \frac{x^2}{m+3\,n} + \cdots\right)}\ldots\ldots(18)$$

By factoring and direct division the following expression is obtained, which gives the limiting value of r as x approaches zero:

$$r = \frac{m}{m+n}\left[1 + \frac{n^2\,x}{(m+n)(m+2\,n)} + \frac{(m^2 + n\,m + 2\,n^2)\,n^2\,x^2}{(m+n)^2\,(m+2\,n)\,(m+3\,n)} + \cdots\right]\ldots\ldots(19)$$

The detention ratio can be evaluated by substituting values of r into the equation

$$d = 1 - r\,x\ldots\ldots(20)$$

which follows from the definitions of d, r, and x. Equations (18) and (20) were used in computing the values of the detention ratio given in Table 2.

Time of Emptying.—The time required to empty a reservoir after the cessation of the uniform flood is found by letting the inflow equal zero in Equation

(8), which then becomes

$$\text{Zero} = B\, h^n\, dt + C\, m\, h^{m-1}\, dh \qquad (21)$$

Transposing,

$$dt = -\frac{C\, m}{B}\, h^{m-1-n}\, dh \qquad (22)$$

Integrating,

$$\int_T^t dt = -\frac{C\, m}{B} \int_H^h h^{m-1-n}\, dh \qquad (23)$$

and,

$$t - T = \frac{1}{m-n}\, \frac{C\, m}{B}\, (H^{m-n} - h^{m-n}) \qquad (24)$$

except when $m - n =$ zero. From Equation (24),

$$t - T = \left(\frac{m}{m-n}\right) \frac{S}{O} \left[1 - \left(\frac{o}{O}\right)^{(m-n)/n} \right] \qquad (25)$$

The total time T' required to empty the basin after the uniform flood ceases is found by letting o equal zero in Equation (25) and therefore

$$T' = \left(\frac{m}{m-n}\right) \frac{S}{O} \qquad (26)$$

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PAPERS

GENERAL WEDGE THEORY OF EARTH PRESSURE

By Karl Terzaghi,[1] M. Am. Soc. C. E.

Synopsis

During the twenty years since the first World War the most important development in earth pressure theory has been the transition from the original theory of Coulomb to a broader concept called "the general wedge theory."

According to this theory the shape of the surface of sliding and the position of the resultant earth pressure are a function of the degree and type of freedom for lateral expansion of the supported earth. Coulomb's theory represents a special case of the general wedge theory. Its validity is limited to the pressure of earth on supports such as retaining walls whose lateral resistance is independent of their amount of yield. In every other case the general wedge theory should be used. The only condition required to make this theory valid is an almost complete mobilization of the shearing resistance of the supported earth within a continuous zone extending from the foot of the bank to its upper surface. This single requirement, however, is sufficient to exclude the general wedge theory (or any other theory involving the condition of incipient failure) from application to the pressure of soft clay or other highly elastic types of soil on the timbering of cuts or tunnels.

General Principles

The essentials of the general wedge theory can be described as follows: If the lateral support of a mass of earth yields, a wedge-shaped body of earth descends toward the yielding support. Before this process begins, however, the weight, W, of the wedge (see Fig. 1), the resultant reaction, F, along the surface of sliding, and the reaction P_a, acting along the back of the lateral support, must be in equilibrium, and the shearing resistance of the backfill must be almost completely mobilized within a continuous zone, extending from the

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Note.—This paper was presented at the meeting of the Soil Mechanics and Foundations Division at Rochester, N. Y., on October 14, 1938. Written comments are invited for immediate publication; to insure publication, the last discussion should be submitted by **February 15, 1939.**

[1] Dr. Ing., Harvard Univ., Cambridge, Mass.

foot of the backfill to its upper surface. This broader concept of the general wedge theory involves no assumption concerning the shape of the surface of sliding and none concerning the location of the center of the resultant earth pressure. The surface of sliding must satisfy the condition that the corresponding reaction, P_a, required to prevent the descent of the wedge, shall be a maximum and that the yield of the lateral support shall be consistent with the conditions of stress and strain within the wedge. An analysis of the manner in which the lateral support yields, and the influence of this yield on the conditions of stress and strain in the wedge, was presented by the writer in 1936.[2] This analysis led to the conclusion that the shape of the surface of sliding and the position of the resultant earth pressure are a function of the degree and type of freedom for lateral expansion of the supported earth. Depending on this degree of freedom, the surface of sliding can assume any shape intermediate between an inclined plane (Fig. 1(*a*)) and a conchoidal surface approaching a vertical asymptote (Fig. 1(*d*)). The center of the resultant earth pressure may be anywhere between 0.33 h and $> 0.5\ h$, and the corresponding distribution of the lateral pressure ranges between hydrostatic and roughly parabolic, as shown in Fig. 1.

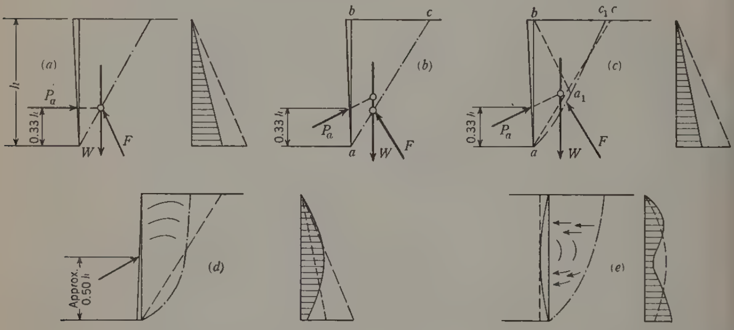

Fig. 1

In the general wedge theory, if the supplementary assumptions are introduced that the surface of sliding is plane and that the distribution of the pressures over both the back of the wall and the surface of sliding is hydrostatic, the result is the well-known earth pressure theory of Coulomb. Analogous to the introduction of these assumptions in the general wedge theory is the assumption that plane sections remain plane, which Navier introduced in the general theory of bending to obtain the simplified theory of bending. If this simplified theory is applied to the bending of short and very thick beams, erroneous results are obtained. Similarly, the theory of Coulomb leads to unsatisfactory results if the conditions of the problem are incompatible with the fundamental assumptions.

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[2] "A Fundamental Fallacy in Earth Pressure Computations," by Karl Terzaghi, *Journal*, Boston Soc. C. E., April, 1936.

In order to define the boundaries between the realm of validity of Coulomb's original theory and that of the general wedge theory, the writer presents herein an analysis of the different conditions that are likely to influence the intensity and the distribution of the active earth pressure. For the sake of brevity the analysis is limited to the investigation of pressures exerted by cohesionless backfills on vertical lateral supports. Neither the influence of surcharges nor that of percolating water on the lateral pressure will be taken into consideration.

Walls Tilting Around Inner Lower Edge

If the back of a wall is perfectly frictionless and if, in addition, the wall yields by tilting around its lower edge, Coulomb's assumptions are fully satisfied and the results obtained by his method are correct. This case is illustrated by Fig. 1(*a*). If the back of the wall is rough, involving an angle of wall friction δ the forces that act on the wedge according to Coulomb's theory are shown in Fig. 1(*b*), and the corresponding distribution of the lateral pressure over the back of the wall is hydrostatic. The forces shown in Fig. 1(*b*) fail to intersect in one point, which is incompatible with the fundamental conditions for the equilibrium of the wedge. This contradiction inherent in Coulomb's theory is due to the fact that the lower part of the real surface of sliding is curved slightly as shown in Fig. 1(*c*). The corresponding distribution of the lateral pressure is also hydrostatic. An accurate computation of the true shape of the surface of sliding and of the true intensity of the lateral pressure for the case illustrated by Fig. 1(*c*) was accomplished by Theodor von Kármán, M. Am. Soc. C. E., in 1926,[3] by J. von Jaky in 1931,[4] and by J. Ohde in 1938.[5] These investigators used different methods, but the results are essentially identical. They demonstrate that the error involved in Coulomb's theory does not exceed several per cent. For practical purposes such an error is without significance.

Similar investigations have been conducted concerning passive earth pressure (resistance of sand against lateral displacement). When applied to a wall with a smooth and vertical back (Fig. 2(*a*)), Coulomb's theory of passive earth pressure is strictly correct. On the other hand, if the theory is applied to a similar wall with a rough back, as shown in Fig. 2(*b*), Coulomb's assumptions concerning the forces that act on the wedge are conspicuously in contradiction to the conditions for the equilibrium of the wedge. Both experiment and the advanced theories revealed by von Kármán[3] and von Jaky[4] demonstrate that the surface of sliding is curved as shown in Fig. 2(*c*). They also demonstrate that the error due to Coulomb's assumptions may exceed 30%, provided the value of the angle of wall friction δ approaches that of the angle of internal friction, ϕ. Hence, if δ exceeds several degrees, it is necessary to modify the theory by assuming that the sliding surface is not a plane. For all practical purposes, one can approximate the shape of the sliding surface by considering the lower portion as a segment of a logarithmic spiral and the upper portion as a straight line rising at an angle of $(45 - 0.5\,\phi)^\circ$ to the horizontal, as shown in Fig. 2(*c*).[5] The position of the center of the spiral must be determined in such a manner that the corresponding value of the passive earth pressure is a minimum. The most convenient method for accomplishing this result is by trial. An accurate method of determining the shape of the surface of sliding was introduced in 1938 by Ohde.[5] However, the application of this method requires considerable time and labor, and the difference between the results thus obtained and those obtained by means of the aforementioned approximate method does not exceed a few per cent. The assumption of a hydrostatic pressure distribution is reasonably correct, provided the wall advances toward the fill by tilting around the lower edge.

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[3] "Über elastische Grenzzustände," by Theodor von Kármán, *Proceedings*, Second International Cong. for Applied Mechanics, Zurich, 1926.

[4] "Die klassische Erddrucktheorie. Vezügyi Közlemenyek," by J. von Jaky, Budapest, 1931.

[5] "Zur Theorie des Erddruckes," by J. Ohde, *Die Bautechnik*, 1938.

Far more important than the errors due to the approximation regarding the shape of the surface of sliding are those due to the uncertainties concerning the angle of wall friction. Thus far the results of pertinent investigations seem to indicate that the angle between the resultant passive earth pressure and the normal to the wall is considerably smaller than the angle of friction between backfill and wall unless the state of stress in the mass of sand subject to the later pressure approaches the point of failure. Due to the important bearing of the subject of passive earth pressure on the stability of bulkheads, further experimental investigations concerning the angle of wall friction are urgently needed.

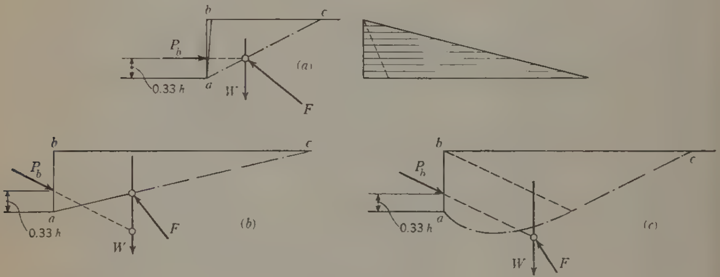

Fig. 2

In the cases illustrated by Fig. 1(*a*) to 1(*c*) the distribution of the active earth pressure over the back of the wall is hydrostatic. The theories which apply to these cases may also be used for computing the active earth pressure on retaining walls or other lateral supports which are capable of yielding indefinitely without being weakened. Large-scale earth pressure tests made by the writer in 1929 demonstrated conclusively that the distribution of the lateral pressure of sand is always hydrostatic after the lateral support has yielded beyond a certain critical distance which depends on the height of the fill and on the character of the backfill.[6]

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[6] "Large Retaining Wall Tests. I. Pressure of Dry Sand," by Karl Terzaghi, *Engineering News-Record*, February 1, 1934.

On the other hand, the capacity of the timbering in a cut to yield under pressure is very limited and in no case can it be expected to reach the value of the yield required to induce a hydrostatic pressure distribution. For this reason it is necessary to consider the influence of a small yield of a lateral support under the conditions associated with excavation and timbering on the intensity and the distribution of the lateral pressure.

TIMBERING OF EXCAVATIONS IN SAND

As early as 1908, the late J. C. Meem, M. Am. Soc. C. E., on the basis of his experiences in subway construction in New York City, maintained that the distribution of the earth pressure over the timbering of cuts is decidedly nonhydrostatic, and that the surface of sliding intersects the surface of the bank at a right angle.[7] His statements were corroborated by similar practical experiences described in 1920 by H. G. Moulton,[8] M. Am. Soc. C. E. While making his large-scale earth pressure tests in 1929 the writer observed that the surface of sliding in the backfill of a sliding wall has an abnormally small radius of curvature and that the center of the earth pressure acts at an elevation of 0.45 h above the base of a backfill with height h.[6] In 1935 the writer demonstrated by theory,[9] on the assumption of a plane surface of sliding, that the center of the active earth pressure may be at any elevation between 0.33 h and 0.66 h above the base of the backfill, the exact position of this point depending on the degree of freedom of the supported earth to expand in a horizontal direction.[2, 9] He also arrived at the conclusion that the type and degree of freedom for lateral expansion of a mass of sand during the process of excavation and timbering is practically identical with that of a backfill behind a wall which yields by slight tilting around its upper edge. In 1938 Ohde published a method for computing the lateral pressure which corresponds to that type of a yield. He found for this case that the center of pressure should be at a height of 0.55 h above the base of the fill and that the lateral pressure is appreciably greater than the Coulomb pressure. The last mentioned fact can be expressed by an equation

$$P_{an} = C_k \, k_a \frac{w_m \, h^2}{2} \qquad \text{(1)}$$

in which P_{an} = horizontal component of the total lateral pressure; w_m = unit weight of the backfill; h = depth of the cut; k_a = hydrostatic pressure ratio for Coulomb conditions; and C_k = the wedge factor, a factor that expresses the influence of the state of strain in the wedge on the total intensity of the lateral pressure.

The value of k_a can be computed by means of Coulomb's theory. It depends only on the angle of internal friction ϕ, and on the angle of wall friction, δ. The wedge factor C_k depends on the state of strain imposed on the wedge by the lateral boundary conditions. It can be computed by means of Ohde's theory, which is based on the assumption that the surface of sliding can be replaced by an arc of a circle. However, the method is far too complicated for universal practical application. For this reason the writer developed a simplified method of computation. It is based on the empirical facts that the surface of sliding intersects the horizontal surface of the bank at a right angle and that the center of the lateral pressure on the timbering of a cut with a depth h in sand is somewhere between an elevation of 0.50 h and 0.60 h above the bottom of the cut. The agreement between Coulomb's and von Kármán's theory of the active earth pressure and between Ohde's rigorous and simplified theory of the passive earth pressure prove that a moderate departure of the assumed shape of the surface of sliding from the real one has little influence on the results. In the case under consideration the writer found that the most convenient substitute for the real surface of sliding is a logarithmic spiral with the polar equation

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[7] "The Bracing of Trenches and Tunnels, with Practical Formulas for Earth Pressures," by the late J. C. Meem, *Transactions*, Am. Soc. C. E., Vol. LX, June, 1908, p. 1.

[8] "Earth and Rock Pressure," by H. G. Moulton, *Transactions*, Am. Inst. of Mining and Metallurgical Eng., 1920.

[9] "Distribution of the Lateral Pressure of Sand on the Timbering of Cuts," by Karl Terzaghi, *Proceedings*, International Conference on Soil Mechanics, Vol. I, Cambridge, Mass., 1936.

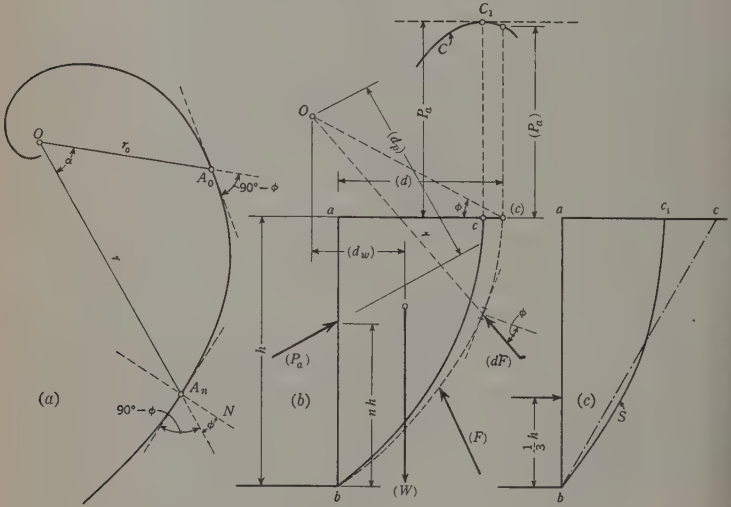


FIG. 3

$$r = r_0\, \epsilon^{\alpha \tan \phi} \quad \text{.......................} (2)$$

in which r = length of a vector through the center of the spiral; r_0 = length of an arbitrarily selected reference vector; ϵ = base of natural logarithms; α = angle between r and r_0, in radians; and, ϕ = angle of internal friction of the backfill. Fig. 3(a) shows such a spiral corresponding to Equation (2). It has the property that a vector r passing through the center of the spiral and any arbitrary point A_n of the spiral intersects the tangent to the spiral in point A_n at an angle $90 - \phi$. As a consequence the angle between the vector r and the normal N to the tangent in point A_n is equal to the angle ϕ of internal friction.

· In order to satisfy the boundary conditions of the problem, the spiral is oriented with reference to the face of the bank $a\ b$, Fig. 3(*b*), in such a manner that the spiral passes through the foot of the bank b, Fig. 3(*b*), and that it intersects the horizontal surface of the bank at a right angle, at an arbitrary distance (d) from the upper rim a. The angle of wall friction, δ, and the elevation $n\ h$ of the center of pressure are assumed to be known. The general wedge theory requires that the frictional resistance of the surface of sliding should be almost fully mobilized over the entire surface of sliding, $b\ c$ in Fig. 3(*b*). As a consequence the reaction, dF on each element of the sliding surface in Fig. 3(*b*) intersects the tangent to the surface of sliding at an angle $90 - \phi$. According to Equation (2) every vector r also intersects the corresponding tangent at the same angle, $90 - \phi$. All the vectors r pass through the center of the spiral (point O, Fig. 3(*b*)). Hence all the reactions (dF) and the resultant reaction, (F), pass through point O. Force W due to the weight of the wedge $a\ b\ c$ passes through the center of gravity of area $a\ b\ c$. Taking moments around the center of the spiral, the following formula is obtained as a condition for the equilibrium of wedge $a\ b\ c$: $(P_a)\ (d_p) = (W)\ (d_w)$;
or—

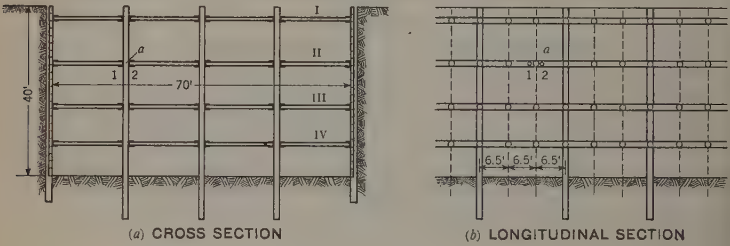
$$(P_a) = (W)\, \frac{(d_w)}{(d_p)} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The remainder of the investigation is practically identical with the graphical solutions of Coulomb's theorem. Several spirals are assumed to intersect the horizontal surface of the bank at different distances (d) from the rim, a, of the cut. In order to facilitate tracing the spirals the writer plots Equation (2) on a piece of cardboard and cuts it out to be used as a pattern for tracing the spirals in the graph. For each one of these spirals he determines the corresponding value (P_a) and plots these values as ordinates above a (c) in Fig. 3(*b*). Thus he obtains curve C. According to the well-known procedure for graphic earth pressure computations, the active earth pressure P_a is equal to the ordinate of the highest point C_1 on curve C and the position of the corresponding surface of sliding is determined by point c beneath point c_1.

One would expect the greatest error involved in the graphical method illustrated by Fig. 3 to occur when the method is applied to the computation of the active earth pressure on a wall, Fig. 1(*b*), which yields by tilting around its lower edge, because in this extreme case the surface of sliding does not resemble a logarithmic spiral. The corresponding numerical values are $n = \frac{1}{3}$ and the wedge factor $C_k = 1.0$. By introducing into his computation the values $\tan \phi = 0.6$ and $\delta = 0$, the writer obtained $C_k = 0.94$ instead of the correct value 1.0. Curve S in Fig. 3(*c*) represents the curved sliding surface for this case and line $b\ c$ is the corresponding surface of sliding obtained by means of Coulomb's theory. In spite of the conspicuous difference between these two lines, the error does not exceed 6%. If the value of n is high, the surface of sliding actually has the shape assumed in the general wedge theory. Therefore the error can be expected to be still smaller. For $\phi = 37°$; $\delta = 26°$; and $n = 0.6$ the computation furnished $C_k = 1.27$ and for $n = 0.50$, $C_k = 1.18$. In general the value C_k increases with increasing values of n, ϕ, and δ.

Field Measurements

In order to check the writer's theory as it applies to the lateral pressure exerted on the timbering of excavations, the Siemens Bau Union, in 1936, measured the pressures in the struts of a 38-ft subway cut in Berlin, Germany. The sides of the cut consisted mostly of sand and fine gravel. Fig. 4(*a*) shows a cross section through the cut, and Fig. 4(*b*) shows a longitudinal section. Before excavation was begun, two outer and three inner rows of I-beams were driven into the ground. In the outer rows the spacing was 6.5 ft, and in the inner rows it was about 20 ft. As excavation proceeded, horizontal boards were inserted between the flanges of the steel beams of the outer rows. The struts were arranged so as to brace the steel beams of the outer rows against one another across the cut. After excavation was finished the lateral pressure of the earth was taken by four sets of struts. In every set the average horizontal distance between the struts was 6.5 ft. The vertical distance between the sets decreased from the top set to the bottom set. Each strut consisted of four pieces bearing against the web of horizontal channels and fastened to the steel I-beams of the inner rows. The distance between the webs of each pair of channels was maintained by means of wooden spacers (*a*, in Fig. 5).



(*a*) CROSS SECTION (*b*) LONGITUDINAL SECTION

FIG. 4

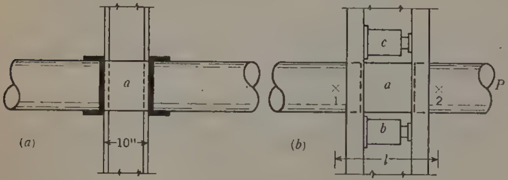


FIG. 5

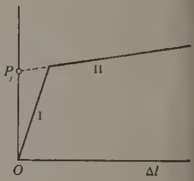


FIG. 6

In order to measure the axial pressure in a strut, two small hydraulic jacks were inserted between the channels (Fig. 5(*b*)) on both sides of the spacer. Then the jack pressure was increased gradually and the corresponding increase, Δl, of the distance l, between points 1 and 2 was measured. The relation between the total jack pressure, P_j, and the corresponding values of Δl are shown in Fig. 6. As long as the pressure exerted by the jacks is less than the pressure in the strut, the jacks merely relieve the pressure in the spacer without changing the pressure in the long sections of the strut.[10] The corresponding increase of the distance between points 1 and 2 is slight, as shown by the steep rise of section I of the $(P_j - \Delta l)$-line in Fig. 6. However, as soon as the jack pressure exceeds the pressure in the struts, the pressure in the long sections of the strut increases and, consequently, the deflectometer readings increase rapidly (see flat section II of the $(P_j - \Delta l)$-line in Fig. 6). The pressure that acted in the strut before the jack pressure was applied is determined by the ordinate of the point of intersection of the vertical axis of the graph in Fig. 6, and the prolongation of section II of the $(P_j - \Delta l)$-line. Fig. 7 is a view of the test arrangement,[10] in which "1" represents the struts, "2" the spacer, "3" the jacks, "4" the extensometer, and "5" the channels.

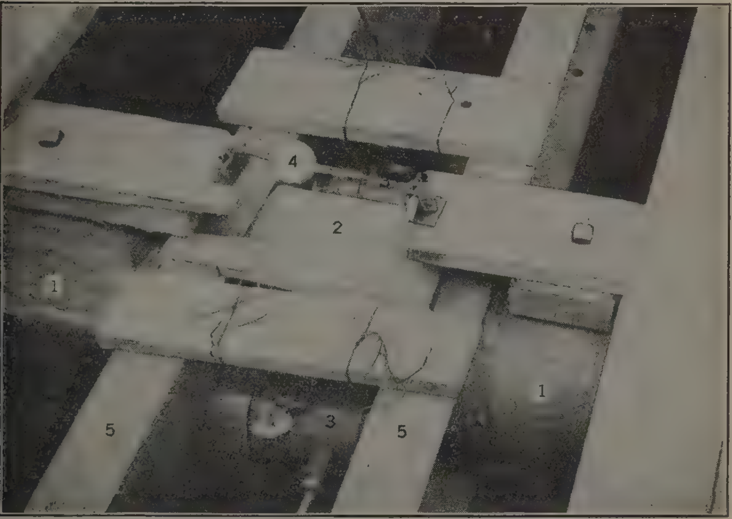


FIG. 7

The measurements were made in the struts at six places (4 to 7, and 9 to 10 in Fig. 8) in a section of the cut 360 ft long. At profiles 1 to 3 the measurements were limited to the top strut and at profile 8 to the second strut from the top. Fig. 9 shows the results of some of the measurements.

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[10] "Mitteilung über die Messung der Kräfte in einer Baugrubenaussteifung," by A. Spilker, *Die Bautechnik*, 1937, Heft 1.

In order to discover whether the frictional resistance of the supported banks had been mobilized, a comparison was made by the writer between the theoretical value and the measured value of the total pressure in the struts at the different sections. For this purpose, Equation (1) was used. The unit weight of the sand[10] was 1.7 tons per cu m, the angle of internal friction ϕ was 37°, and the angle of wall friction δ was 26°. The values of the angles ϕ and δ undoubtedly were estimated and not measured. Due to the absence of fine material, the cohesion was negligible.

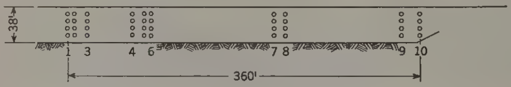

FIG. 8

With the numerical values mentioned, Coulomb's theory yields the value $k_a = 0.204$; and, from the general wedge theory, $C_k = 1.22$. The computations, according to the general wedge theory, yield a theoretical value for the total lateral pressure which is an average of 25% greater than the value obtained from the measurements. These findings seem to indicate that the angle of internal friction of the supported material was appreciably greater than the estimated value of 37° (at least 41°), and that the shearing resistance of the supported banks was almost fully mobilized, satisfying the only fundamental requirement of the general wedge theory.

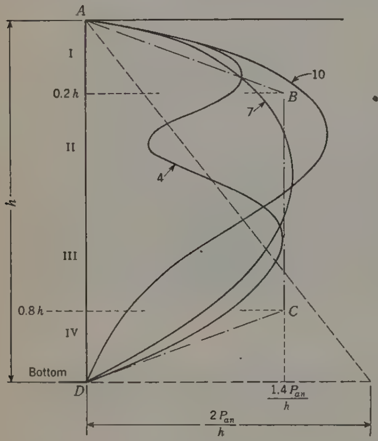

FIG. 9

According to the general wedge theory for the pressure of sand on timbering in excavations, the distribution of the pressure over the lateral support should be more or less parabolic, having a maximum pressure at approximately half the depth of the cut. This conclusion is fully substantiated by the results of the measurements shown in Fig. 9.

The data shown in Fig. 9 also indicate the magnitude of the deviations of the true pressure distribution from the statistical and theoretical average. According to the geological profile of the cut, the supported banks are fairly uniform. Hence, it seems most likely that the observed variations are essentially due to accidental variations in the procedure followed in constructing the timber work.

Foremost among these variations is undoubtedly the variation in the force with which the wedges were driven. Another cause may be the local variations in the density of the supported material, because such variations are likely to induce local arching. This might account for the unusual shape of pressure curve 4 in Fig. 8. On the other hand, the location of the center of the lateral earth pressure was quite remarkably consistent. It varied between the relatively narrow limits of $0.53\ h$ and $0.60\ h$ above the bottom of the cut. The most frequent values were between $0.53\ h$ and $0.55\ h$.

These variations must be considered inevitable. For this reason the writer has always insisted that ample margin be provided for possible scattering. In Fig. 9 this margin is indicated by the trapezoid $A\ B\ C\ D$. If h indicates the depth of the cut and P_{an} the horizontal component of the total lateral pressure on the timbering, the width of the trapezoid is equal to $1.4\ \frac{P_{an}}{h}$. Fig. 9 shows that the observed variations are at least as large as those which are taken care of by the suggested margin.

FLEXIBLE BULKHEADS

Another problem of outstanding practical importance concerns the earth pressure exerted on flexible bulkheads. The total earth pressure can be expected to be equal to that which acts on the back of a yielding, non-flexible retaining wall; but the distribution of the pressure is quite different, because the lateral bulging induces a concentration of the pressures in the vicinity of the upper and the lower support at the expense of the pressure on the middle, yielding part (see Fig. 1(*e*)). This fact seems to have been noticed for the first time by Danish engineers. When examining some of their century-old bulkheads they found out by computation that these bulkheads could not possibly stand the lateral pressure of the supported earth unless the distribution of the pressure is similar to that shown in Fig. 1(*e*).[11] This observation led to a radical revision of the Danish specifications for the design of bulkheads.[12] In a case of sheet-pile computations which came to the attention of the writer, the value for the maximum bending moment obtained by means of the Danish method was about one half the value obtained on the basis of Coulomb's theory. The economy inherent in the application of this procedure is obvious.

In 1936 P. R. N. Stroyer published the results of experiments with flexible steel bulkheads supporting dry sand backfills 3 ft deep.[13] The results of these tests can also be explained only on the assumption that the distribution of the lateral pressure over the bulkheads was such as that shown in Fig. 1(*e*), which in turn requires intense "arching" within the sand behind the bulkhead. Some engineers still seem to believe that the arching effect is a temporary phenomenon, destined to disappear in time. The Danish experience mentioned previously represents one of many empirical facts which invalidate this pessimistic opinion. Some of the Danish bulkheads were exposed for centuries to the pounding of waves, and for decades to heavy traffic, without the distribution of the lateral pressure becoming "hydrostatic."

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[11] "Calcul des murs de quai," by G. Schønweller, *Proceedings*, World Eng. Cong., Rept. No. 416, Tokyo, 1929.

[12] "Forelobige Regler for Beregning af Jaernbetonkonstruktioner i Vandbygning," published by the Dansk Ingeniörforening (no date); see also "Ztschr. des ständ. Verb. intern. Schiffahrtskongr.," No. 7, January, 1929.

[13] "Earth Pressure on Flexible Walls," by P. R. N. Stroyer, Paper No. 5024, *Journal*, Inst. C. E., November, 1935 (discussion of this paper by the writer, January 1936).

The arching between the upper and the lower support of flexible bulkheads eliminates Coulomb's theory from application to bulkhead problems. The influence of the lateral deflection on the location of the center of pressure is not yet known. Undoubtedly, it depends not only on the type of backfill and on the flexibility of the wall, but also on the distance through which the upper and the lower support of the bulkhead are capable of yielding and on the density of the backfill.

Because of the outstanding practical importance of the problem, the writer advised the Soil Mechanics and Foundations Division of the Society to include it in its research program. The variables to be considered in such an experimental investigation are: (*a*) Degree of fixation of the lower part of the bulkhead, (*b*) position of the upper support (anchorage) with reference to the top of the backfill, (*c*) yield of the anchorage, (*d*) deflection of the flexible part of the bulkhead with reference to a plane through the upper and lower supports, and (*e*) the relative density of the backfill. Investigations of this kind could be made in a well-equipped earth-pressure laboratory at a small expense for additional special equipment. It would also be advisable to include in the program the measurement of the deflection of full-sized bulkheads, to compute the corresponding bending moments and to supplement the results of the measurements by determining the physical properties of the backfill.

PRESSURE EXERTED BY COHESIVE EARTH

For cohesive soils the general wedge theory is based on Coulomb's equation for the shearing resistance, s, of cohesive soils. In this empirical equation,

$$s = c + p \tan \phi \ldots\ldots\ldots\ldots (4)$$

c is Coulomb's cohesion value, p is the unit pressure normal to the surface of sliding, and ϕ is Coulomb's angle of internal friction. However, there are many conditions under which Equation (4) does not give even approximate values. Hence, at the very best, it should be considered a crude approximation to reality. If used without careful discrimination, it can be seriously misleading.[14] For cohesive soils, the influence of the type of yield of the lateral support on the location of the center of pressure, and on the distribution of the lateral pressure, is still more important than it is for cohesionless ones. If the lateral support yields by tilting around its lower edge, one must anticipate a hydrostatic pressure distribution, beginning at some depth below the upper edge. Above this depth the soil can stand without any lateral support. On the other hand, if the lateral expansion of the wedge increases with depth (as it does during the excavation of a timbered cut) the heavy pressure acts on the upper part of the timbering, whereas the lowest part of the sides can be left without any support. These facts can be demonstrated easily by means of the theory of arching.[9] By analogy with reference to the theory of cohesionless soils, one may also conclude, in accordance with experience, that the section through the surface of sliding should start with a vertical tangent at the surface of the bank and descend toward the foot of the bank with a very marked curvature. Hence, there is no contradiction between the general wedge theory and experience, provided the yield of the lateral support is sufficient to mobilize the major part of the shearing resistance of the supported banks.

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[14] "Einfluss des Porenwasserdruckes auf den Scherwiderstand der Tone," by Karl Terzaghi, *Deutsche Wasserwirtschaft*, 1938, Heft 8/9.

This mobilization of the shearing resistance, however, can only be anticipated in comparatively dense strata of sandy soils. If the soil is fairly elastic, as is the case for plastic or soft clay, the lateral expansion of the sides of the cut during excavation and timbering can scarcely be expected to result in mobilizing more than a fraction of the shearing resistance of the supported material. In this case the intensity of the lateral pressure of the earth must be greater than the value obtained by means of the general wedge theory. The importance of the difference depends on the elastic properties of the supported soil, the distance through which the material can yield during the process of excavation and timbering, the state of stress that existed in the earth before excavation was begun, and on the time factor. Because of the difficulties connected with evaluating these many variables, and the complicated elastic properties of cohesive soils, the prospects of predicting the lateral pressure successfully by theory are slight. A substantial advancement of pertinent knowledge can be expected only from actual measurement of the pressure in the braces in cuts through different clay soils, and subsequent correlation of the results with the physical properties of the materials encountered in the cuts in which the measurements were made. A satisfactory method for making such measurements is described herein, and illustrated by Figs. 5 to 7. If steel beams are used to support the sides of a cut, the pressure in the struts can be determined by means of a strain gage such as the Whittemore strain gage. The reference points should be placed and their distance recorded before the beams are lowered into the cut.

On account of the appalling scarcity of published information concerning the pressure of clay on timbering in excavations, the results of every reliable observation regarding this subject should be considered important. Therefore the writer has requested the Soil Mechanics and Foundations Division to collect such data and to promote the measurement of the pressure on the timbering in excavations wherever there is an opportunity to do so. The results thus obtained could not fail to produce a radical revision of the specifications for the computation of earth pressure under such conditions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

SEWAGE DISPOSAL PROJECT OF BUFFALO, NEW YORK

By Samuel A. Greeley, M. Am. Soc. C. E.[1]

Synopsis

The purpose of this paper is to make of record, with proper explanation, some of the engineering data accumulated during the design and construction of the sewage disposal project of Buffalo, N. Y. During the three years 1936 to 1939, the Buffalo Sewer Authority completed the construction of intercepting sewers, sewage treatment works, and storm-water relief sewers, at a total project cost of $15,000,000. The sewage disposal works were placed in operation toward the end of June, 1938.

General Statement

The Buffalo Sewer Authority was created by a special act of the Senate and Assembly of the State of New York and became a law April 8, 1935. The five members of the Authority were first appointed in June, 1935, and later in the year, on December 7, the Authority appointed the organization which was to serve as designing and supervising engineers. Meanwhile, on June 18, the Authority made application to the Federal Emergency Administration of Public Works for a loan and grant, in the amount of $15,000,000, to aid

> "* * * in financing the construction of a system of trunk, intercepting and connecting, lateral and outlet sewers, storm water drains, pumping and ventilating stations, disposal or treatment plants or works, and other appliances and structures to provide an effectual and advantageous means for relieving Niagara River, Buffalo River and Lake Erie from pollution by the sewage and waste of the City of Buffalo and relieving the City of Buffalo from inadequate sanitary and storm water drainage and for the sanitary disposal or treatment of the sewage of the City of Buffalo."

The estimates of cost in this application included construction work in the sum of $13,200,000, and a sum of $1,800,000 to cover preliminary expenses, land and right of way, engineering, interest during construction, legal, and administrative costs. The proposed storm-water relief sewers were for the Bird Avenue Sewer District only.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February 15, 1940.**

[1] Greeley & Hansen, Chicago, Ill.

The first investigation of the sewage disposal problem of Buffalo was made between 1909 and 1918 by the International Joint Commission in connection with an investigation of the pollution of boundary waters. The conclusions stated in their final report are briefly summarized as follows:

> "It is advisable to consider the Niagara and Detroit Rivers as a class by themselves, so far as remedial and preventive systems are concerned. As has been stated, the most serious condition existing is the bacterial pollution of these streams. To remedy this evil, sewage treatment should be applied in connection with dilution so far as is necessary to bring their waters to the standard mentioned—a mean annual cross-sectional average of B. coli not exceeding 500 per 100 c.c. This necessary sewage purification can be effected by fine screening or sedimentation, and when necessary by chemical disinfection, at a cost which will impose no unreasonable burden upon the urban community responsible for the present pollution."

Furthermore, it has been clear for a number of years that the waterways within the city limits of Buffalo have been grossly polluted, including Buffalo River, Buffalo Harbor, and Black Rock Harbor. The function of the sewage disposal part of the project, therefore, is threefold, as follows:

(*a*) To relieve Buffalo River and its immediate tributaries from undue pollution;

(*b*) To relieve Buffalo and Black Rock Harbors from undue pollution; and,

(*c*) To remove contamination from Niagara River and to reduce the bacterial content of its waters in order to avoid placing an undue pollutional load on water purification works of communities below Buffalo which derive their water supply from the Niagara River.

A number of the existing sewers had also been overloaded and much in need of relief. As this was especially true of the existing sewers in the Bird Avenue District, it was decided to include storm-water relief sewers for this sewer district in the project.

Action for undertaking the project was stimulated by the orders of the New York State Department of Health, issued in March, 1934, which required that during 1935 the City of Buffalo must engage engineers, take steps to acquire land on which to construct sewage treatment works, submit plans for the Bird Avenue storm-relief drains, and make a satisfactorily comprehensive general plan for interception and treatment of the entire sanitary sewage of the city. Detailed plans and specifications for a substantial part of the work were required to be completed by April 1, 1936, and construction work started on or before July 1, 1936. The order required that the treatment works be completed and placed in operation on or before October 1, 1937.

Actual construction work was begun on a part of the Bird Avenue storm-water relief drains in December, 1935, and a contract for borings, which ultimately required the expenditure of about $38,000, was started in January, 1936. The first major construction contract went into actual construction in June, 1936, under bids which had been advertised on April 23, 1936. An average monthly rate of construction was maintained during twenty-two consecutive months (from August, 1936, through May, 1938) of $547,000.

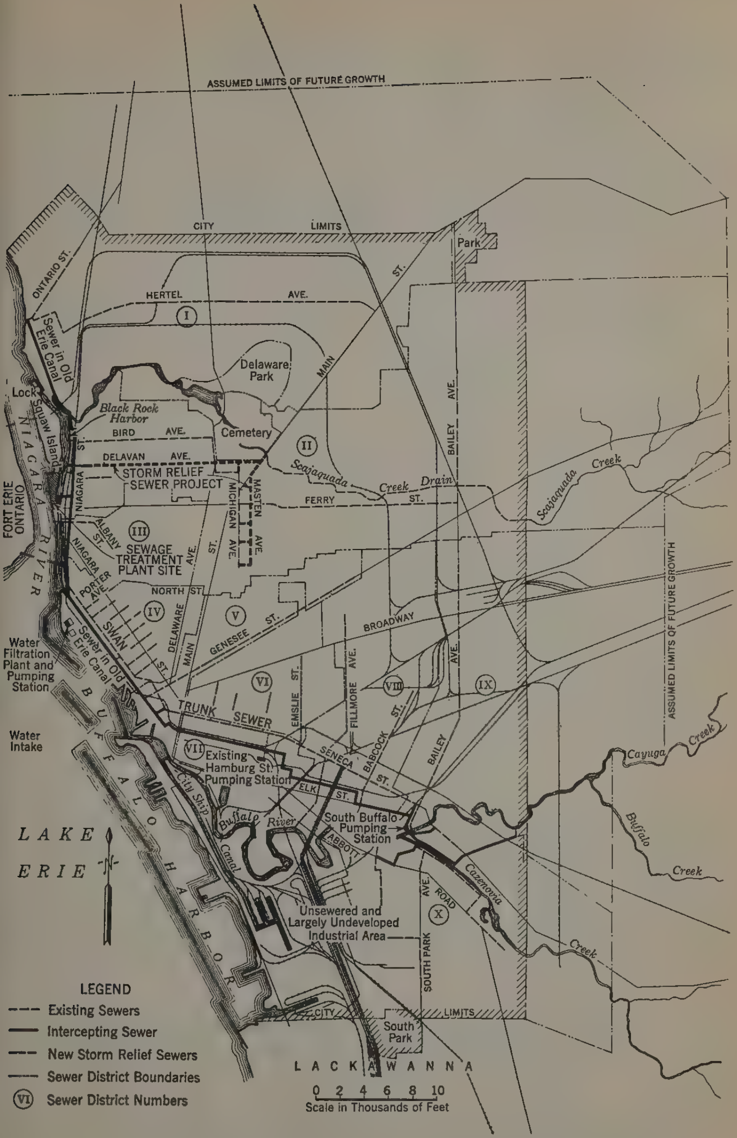


Fig. 1.—General Plan of Sewage Disposal Project

The City of Buffalo, New York

Buffalo is situated along the eastern shores of Lake Erie and the Niagara River (see Fig. 1). The southern part of the city is cut by the Buffalo River which extends from the harbor in a southeasterly and easterly direction. Much of the area tributary to Buffalo River is occupied by industries, and this is the principal manufacturing district of the city. Scajaquada Creek flows in an east and west direction through the northerly part of the city and is used to form Park Lake in Delaware Park. The northerly part of the city is much more residential in character. The population of Buffalo by the United States Census Bureau, and its estimated probable growth, is as follows:

Year	Population	Year	Estimated population
1840	18,213	1940	670,000
1850	42,261	1950	765,000
1860	81,129	1960	860,000
1870	117,714	1970	955,000
1880	155,134	1980	1,050,000
1890	255,664	1985	1,100,000
1900	352,387		
1910	423,715		
1920	506,775		
1930	573,076		

The total area in 1930 was 26,980 acres and the density of population was 21.3 persons per acre. It was thought likely that the area of the city might be extended somewhat to the north and east in the distant future with the larger populations, but this does not influence the plans. The length of the city in a north and south direction is 8.6 miles and the width in an east and west direction is about 5 miles.

The average temperature and precipitation at Buffalo are indicated in Table 1 which is a summary of data from the U. S. Weather Bureau. The Weather Bureau also reports that the prevailing winds are southwesterly except during November, December, and January, when westerly winds prevail.

TABLE 1.—Meteorological Data; Buffalo, N. Y.

Description	January	February	March	April	May	June	July	August	September	October	November	December	Total
Temperature*	25.4	24.0	31.8	42.5	54.0	64.5	69.9	68.7	62.6	51.4	39.4	29.4	47.0
Precipitation†	3.23	2.74	2.68	2.53	2.85	2.88	2.99	2.81	2.88	3.20	2.98	3.21	34.98
Prevailing wind‡	W	SW	SW	SW	SW	SW	SW	SW	SW	SW	W	W	SW

* Average temperature, in degrees Fahrenheit. † Average total precipitation, in inches. ‡ W = westerly; SW = southwesterly.

Existing Sewers

Practically all the sewers of Buffalo are on the combined plan, their total length being about 710 miles. The first sewers were built prior to 1850;

during the 5-yr period prior to 1855, 23 miles were constructed; and in the 5-yr period prior to 1895, 139 miles were built. Thus, the existing sewers of Buffalo have been in service, in general, for many years. At present (1939) there are about 62 outlets, all of which have now been connected to the new intercepting sewers.

The city is divided topographically into two parts, one draining naturally to the Buffalo River and the other directly to Black Rock Harbor and the Niagara River. With a few exceptions, the main drainage system has developed along these natural lines. In the area north of the lower part of the Buffalo River, a system of canals formerly connected with the old Erie Canal. Some of the city sewers discharged into these canals, which have been covered over and serve as storm overflow sewers. Except in the case of District VII (see Fig. 1), the sewers of Buffalo are on the so-called combined system, with sewage and storm water carried in the same sewers. In District VII the dry-weather flow is confined in part to a separate system of sewers and conducted to the Hamburg Street Pumping Station. This station contains a steam power plant with three horizontal, centrifugal pumps, driven by steam engines. It was formerly operated in conjunction with a refuse incinerator, which has not been used in regular operation since before 1929.

Conditions in the harbor, the canals, and the Buffalo River became objectionable at an early date, and to alleviate this pollution, the so-called Swan Trunk Sewer was built in 1883. This sewer, with a later extension, runs roughly parallel to the Buffalo River and the harbor from Bailey Avenue to Albany Street, where it passes under Black Rock Harbor to the Niagara River. It is 8 ft in diameter for most of its length, and was designed as an intercepting sewer to receive the dry-weather flow from all the area north of Buffalo River and south of the Bird Avenue sewer district. It also receives the discharge of the Hamburg Street Pumping Station. In later years it has been much overloaded and, especially near the upper end, overflows have been frequent or sometimes continuous during dry weather. The capacity of this sewer is utilized in the new intercepting sewer system.

The population and industries adjacent to Buffalo River have continued to discharge their sewage into the sluggish waters of the river. Nearly all the large industries along Buffalo River have private sewers directly to the river.

There are several small areas, chiefly in small separate municipalities, outside the present city limits which discharge sewage into the Buffalo sewers, amounting in the total to about 1,000 acres, with a population of about 13,000.

The characteristics of the major sewer districts of Buffalo are as shown in Table 2.

WATERWAYS

The waterways of Buffalo pertinent to the sewage disposal project are (see Fig. 1): Niagara River, Buffalo Harbor, Black Rock Harbor, Buffalo River, Cazenovia Creek, and Scajaquada Creek.

Niagara River.—The drainage area of the Niagara River, at the outlet of Lake Erie, is about 175,000 sq miles. Based on records of the U. S. Lake Survey, the average discharge is 203,000 cu ft per sec. A discharge of 176,000 cu ft per sec is exceeded 90% of the time. The minimum monthly average discharge of record (142,000 cu ft per sec) occurred in February, 1934; and the maximum discharge (251,000 cu ft per sec) occurred in June, 1876. Thus the discharge is relatively large and uniform.

The elevation of the water surface of the Niagara River opposite the sewage treatment plant on Bird Island covers quite a range. It is estimated that an elevation of − 9.7 (Buffalo City Datum) is exceeded on the average for 99% of the time and an elevation of − 5.7 is exceeded for only 1% of the time. Extraordinary fluctuations in the level of Lake Erie cause momentary peaks in the water level in the river at Bird Island much higher than the foregoing. Thus an elevation of + 2.0 is reached, on the average, once in three years, and an elevation of + 5.0 has an estimated frequency of occurrence of once in fifty years.

TABLE 2.—CHARACTERISTICS AND ESTIMATED AVERAGE SEWER QUANTITIES; MAIN SEWER DISTRICTS

District No. (see Fig. 1)	District	ESTIMATED POPULATION, IN THOUSANDS		AREA, IN ACRES		ESTIMATED AVERAGE SEWAGE FLOW			
		1930	1985	1930	1985	1930		1985	
						Mgd*	Gcd†	Mgd*	Gcd†
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
I	Ontario-Hertel	91.2	232.5	4,470	6,920	17.1	187	37.4	161
II	Bird Avenue	159.7	304.5	5,770	8,760	24.4	154	40.4	133
III	Albany Street	32.9	50.0	860	860	3.4	103	5.4	108
IV	Lake Shore	31.9	46.0	730	730	5.1	160	9.0	196
V	Genesee Street	33.0	40.5	800	800	9.4	285	10.2	252
VI	Michigan-Emslie	55.3	68.7	1,060	1,060	10.3	186	13.8	201
VII	Hamburg Street Pumping Station	20.3	14.0	760	760	7.8	384	8.1	578
VIII	Fillmore-Babcock	65.3	77.2	2,365	2,365	20.8	318	25.0	324
IX	Bailey Avenue	40.2	150.8	2,145	4,830	10.5	261	28.6	190
X	South Buffalo	43.2	115.8	2,410	4,140	13.3	308	27.3	236
	Parks, etc.	....	....	1,220	1,220	....	....	....	....
	Water area	....	....	1,840	1,840	....	....	....	....
	Other non-sewered areas	....	....	2,550	2,550	....	....	....	....
	Totals	573.0	1,100.0	26,980	35,835	122.1	213	205.2	186

* Mgd = million gallons daily. † Gcd = gallons per capita per day.

Below Buffalo, the Niagara River is used as a water supply by Tonawanda, North Tonawanda, Lockport, Niagara Falls, Youngstown, and Fort Niagara, in New York, with a total population of about 150,000, and by the cities of Fort Erie and Niagara Falls in Canada. There is an increasing use of the water for recreation, including boating, fishing, summer homes, and some bathing, mainly limited, at present, to the Canadian shore and to the westerly shore of Grand Island. The average number of B. coli per cubic centimeter at the Niagara Falls water-works intake was 36.7 in 1936 and 27.5 in 1937, with individual samples as high as 240. The analyses of the Sewer Authority laboratory show as great a concentration as 2,400 B. coli per cu cm in the most contaminated sections of the river downstream from Buffalo. The lowest content of dissolved oxygen found during the river surveys of the Buffalo Sewer Authority in July, 1936, was 6.0 ppm.

Buffalo Harbor.—Buffalo Harbor is made artificially by breakwaters in Lake Erie. Little sewage is discharged directly into Buffalo Harbor and it has not been an important factor in the design of the project.

Black Rock Harbor.—On the other hand, Black Rock Harbor has received a considerable quantity of sewage and was seriously polluted. In fact, it is a waterway connecting the New York State Barge Canal with Buffalo Harbor and is relatively long and narrow, having the shape more of a canal than of a harbor. Its length is 16,500 ft, the area of an average cross section is 8,800 sq ft, its surface area is 8,100,000 sq ft, and it has a liquid volume at mean depth of 140,000,000 cu ft. Some circulation (about 100 cu ft per sec) is maintained by the operation of a small outlet at the lower or northerly end of the harbor and by operation of the lock. Variations in the level of Lake Erie also have a marked effect in mixing relatively unpolluted lake water with the harbor water, especially in the upper or southerly end.

The principal source of pollution of Black Rock Harbor was the Bird Avenue outlet which discharged about 25.0 mgd of sewage into the harbor about 3,000 ft south of its lower end. This is about 20% of the sewage of the entire city. An indication of the effect of this pollution is given by the data in Table 3.

TABLE 3.—ANALYSES OF BLACK ROCK HARBOR WATER, FROM INDIVIDUAL GRAB SAMPLES

Date	At Ferry Street		At North End	
	Dissolved oxygen, percentage of saturation	5-day B.O.D., in parts per million	Dissolved oxygen, percentage of saturation	5-day B.O.D., in parts per million
1936				
June 25	78	1.7	0	28
July 29	79	15.0	7	16
August 25	69	4.3	0	22
September 30	51	2.8	0	31
December 3	59	8.1	4	28
1937				
January 5	32	4.3	12	30
February 2	44	11.0	19	19
March 9	60	4.6	13	3
April 6	32	5.2	0	30
May 5	10	5.9	0	39
June 2	16	2.3	0	22
July 7	66	1.3	0	6
August 3	62	....	0	5.4
September 8	40	2.6	0	25
October 5	61	1.0	0	25
November 9	28	....	0	....
December 7	47	4.6	0	39

Buffalo River and Cazenovia Creek.—Buffalo River and its main tributary, Cazenovia Creek, have a drainage area of 436 sq miles. There is no continuous record of gagings of the quantity of water flowing in Buffalo River. However, the U. S. Geological Survey has a record of routine gagings on Cattaraugus Creek, the drainage area of which is adjacent to that of Buffalo River and includes 488 sq miles. Using the Cattaraugus Creek gagings as a basis, the discharge of Buffalo River has been computed as follows:

Period	Discharge, in cubic feet per second
Annual average	650
Average from May 1 to October 31	355
Annual median	340
Median from May 1 to October 31	180

A discharge of 100 cu ft per sec is exceeded 90% of the time for the year as a whole, and for the period from May 1 to October 31, a discharge of 80 cu ft per sec is exceeded 90% of the time.

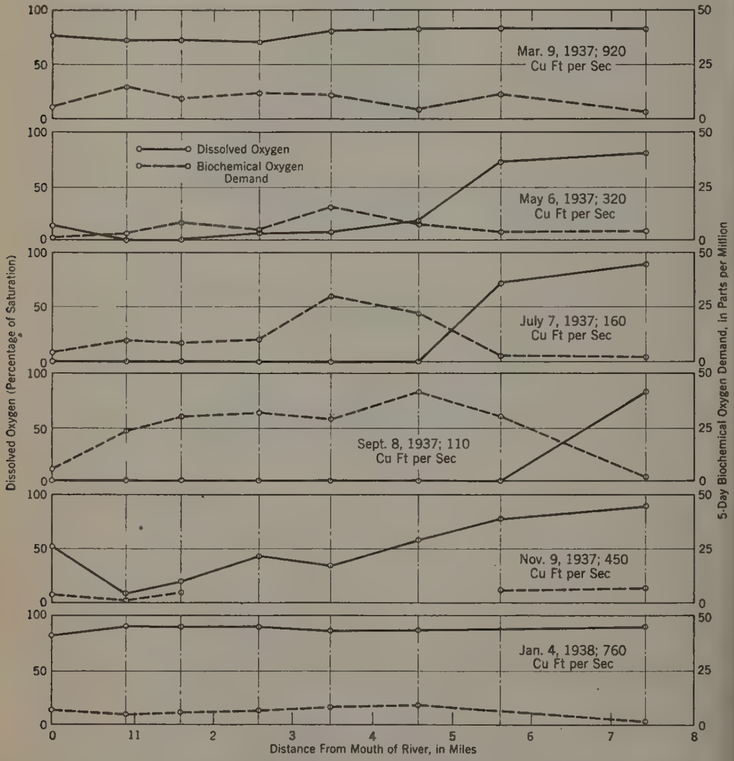


FIG. 2.—CONDITIONS IN BUFFALO RIVER

Fig. 2 shows the dissolved oxygen and the 5-day bio-chemical oxygen demand for five typical river surveys during 1937. The river discharges indicated were determined by actual gagings on the dates of the surveys. During the surveys of May, July, and September the dissolved oxygen was practically exhausted.

Scajaquada Creek.—Flowing through the northerly section of the city into Black Rock Harbor, Scajaquada Creek has a drainage area of 24 sq miles. Where it flows through Delaware Park, it has been dammed to form a small lake. Due to the pollution of waters of the creek, sludge deposits in this lake have caused objectionable odor conditions in the past. The section of the creek between the east city limits and Main Street has been enclosed in a large reinforced concrete drain.

ENGINEERING AND LABORATORY WORK

Much field engineering work was required to be done promptly, including surveys for design and for the acquisition of land and rights of way, investigations of the best location for intercepting sewers, the supervision of borings, examination and gaging of existing sewers, and planning and organizing a laboratory. Much useful work was accomplished through the laboratory, in determining sewage quantities and characteristics and the condition of the water in many of the waterways. Studies and reports were made on most of the larger industries. Investigation of the industrial sewages of Buffalo was a major item of the engineering and laboratory work.

During two periods in 1936 (May 14 to 27 and August 5 to 22), and one in 1937 (July 13 to August 28), several of the large sewers were gaged and sampled to determine the dry-weather characteristics of the sewage. The 1937 study was the most extensive and all the following data refer to it. Each sewer was sampled for a period of 8 to 10 days. Chlorine demand was determined on separate samples collected at bi-hourly intervals. Bacterial analyses were made on two samples daily. All other determinations were made on 24-hr composites, built up from samples taken at 15-min intervals and proportioned according to sewage flow. Table 4 gives the results of this work. The sewage

TABLE 4.—SUMMARY OF 1937 STUDIES OF SEWAGE CHARACTERISTICS BASED ON GAGINGS AND ANALYSES

Sewer	Districts (see Fig. 1)	Tributary area, in acres	Tributary population	Average sewage flow, in million gallons daily	Analytical Results, in Parts per Million						Potential of hydrogen, pH
					Total solids	Suspended solids	Volume of suspended solids	5-day B.O.D.; 20° C	Alkalinity	Chlorine demand	
Bailey Avenue	IX	2,100	40,000	9.95	594	98	68	132	219	3.6	7.4
Babcock Street	VIII‡	400	2,000	4.70	1 930	472	375	498	296	28.4	8.8
Fillmore Avenue	VIII‡	1,350	53,000	9.30	661	183	146	225	207	10.5	7.1
Emslie Street	VI‡	550	32,000	6.10	639	186	153	238	180	13.4	7.0
Swan Trunk*	IV to IX	7,200	235,000	49.41	596	171	134	168	190	8.1	7.1
Albany Street	III	860	33,000	3.75	....	220	....	205	....	5.7	....
Bird Avenue	II	5,340	143,000	24.53	533	145	101	162	177	4.2	7.1
Totals†		13,400	411,000	77.69	....	....	...	....	....	....	....
Estimate for the entire city		21,370	573,000	110	....	153	....	148	....	7.3	....

* This sewer includes the flow from all the first four. † The total includes only the last three items—see preceding footnote. ‡ In part.

quantities were determined from measuring the head on weirs built in the sewers.

During 1936, investigations were made as to the effect of storms on the quantity of suspended solids in the sewage, with special attention to the characteristics of the first flushings, which would be taken to the treatment works. In all, some 150 samples were collected, in twelve studies.

The effect of storm runoff on suspended solids is indicated by the study of the Bird Avenue Sewer, results of which are given in Table 5.

TABLE 5.—EFFECT OF STORM RUNOFF ON SUSPENDED SOLIDS; BIRD AVENUE SEWER

(*a*) AUGUST 19, 1936; TOTAL RAINFALL, 0.55 IN.			(*b*) SEPTEMBER 23, 1936; LOCAL SHOWER					
Time (p.m.) (1)	Sewage flow, in million gallons daily (2)	Suspended solids, in parts per million (3)	Time (p.m.) (1)	Sewage flow, in million gallons daily (2)	Suspended solids, in parts per million (3)	Time (p.m.) (1)	Sewage flow, in million gallons daily (2)	Suspended solids, in parts per million (3)
5:30	25.3	208	7:30	28.5	....	9:00	38.0	532
6:30	120.6	964	7:50	32.8	204	9:15	34.7	600
7:30	104.0	1,220	8:05	41.9	132	9:30	33.0	300
8:30	56.7	696	8:15	46.2	508	9:45	32.8	252
9:30	37.6	364	8:30	46.6	704	10:15	31.0	208
10:30	28.6	172	8:45	43.1	844	....	....	....

Monthly surveys of Niagara River have been made since June, 1936. During the summer months, when the use of a boat was possible, samples were taken at six different cross sections of the river between the site of the treatment plant and the lower end of Grand Island, immediately upstream from the intake of the Niagara Falls water works. About forty stations were sampled at these times. During winter months, samples were obtained only from the two Grand Island bridges, a total of thirteen stations. Analyses were made of each sample to determine the numbers of B. coli and of total bacteria on agar-agar at 37° C. On a few occasions, determinations were also made of dissolved oxygen (D.O.), bio-chemical oxygen demand (B.O.D.), solids, nitrogen, oxygen consumed, alkalinity, and chlorides.

Daily bacterial analyses of the raw water at the Niagara Falls filter plant have been made for the Buffalo Sewer Authority since May, 1936. Two studies were also made to determine the hourly variations in bacterial content of the river water at this point.

Monthly samples from about ten points along the Buffalo River and Cazenovia Creek have been collected and analyzed since April, 1936. The stream discharge and temperature are determined and the following analyses made: Potential of hydrogen, dissolved oxygen, bio-chemical oxygen demand, total solids, suspended solids, oxygen consumed, total nitrogen, chlorides, alkalinity, 37° bacteria, and B. coli. Samples at special locations and for special purposes are collected at various times.

On each of the Buffalo River surveys samples were also collected and analyzed from the following points in the harbors: City Ship Canal at Michigan Avenue, mouth of Buffalo River, Black Rock Harbor at Ferry Street and at Bridge Street.

Occasional studies have been undertaken to determine conditions in the Scajaquada Drain and in Park Lake, with particular attention to sludge deposits and the rate of digestion. Many other more or less extensive investigations have been made covering the chlorine demand of the sewage, hourly variations in sewage characteristics, analyses of materials used in construction, etc.

Industries

Buffalo has a relatively large number of industries which produce sewage. Many of these are situated in the southerly part of the city which drains naturally to Buffalo River. Some of the larger plants discharge directly into Buffalo River and do not use the sewers. The chemical industries form an important group, including seven major plants, among which are those of the General Chemical Company and the National Analine and Chemical Company. There are two relatively large coal coking plants, one of which has been closed. There are twenty-four meat packing plants, eleven breweries, fifteen dairies, eight oil handling and refining plants, five rubber reclaiming plants, and five tanneries.

Many of these industrial establishments have been surveyed and the quantities and characteristics of the sewage determined by gagings and analyses. It has been considered that the sewage from some of the industries is not likely to be readily amenable to treatment. It has thus been desirable to undertake a cooperative program with the industries for the removal of pollution from the Buffalo River and from the sewers of the city. This program has proceeded on a satisfactory basis. Properly, it has not been developed until the later steps of construction and the early steps of operation of the treatment plant.

Storm-Water Relief Sewers

Storm-water relief sewers were planned to serve the Bird Avenue sewer district which has an area of 5,490 acres. The main sewer for this district was inadequate throughout its length, especially the section above Main Street, which has capacities, in its several sections, of only 166 to 665 (gcd), based on estimated 1985 populations. Overflows had been built from this sewer into the Scajaquada Drain, a large sewer which, more or less, is parallel to the Ferry Street sewer to the north. This caused the pollution of Park Lake in Delaware Park. Several lateral sewers in this area were also of insufficient capacity, especially that in Michigan Avenue.

Storm-water relief sewers were planned for a rate of rainfall estimated to occur once in ten years. A runoff coefficient of 0.60 was used for the area within the city limits and of 0.22 for rural areas outside of Buffalo. The size of the necessary relief sewers ranged from 3.5 to 12.0 ft in diameter. The total cost of the storm-water relief sewers was $2,330,000. Plans and specifications for this part of the project were prepared by the City Sewer Department.

INTERCEPTING SEWERS

Intercepting sewers were designed to serve a future population of 1,100,000. The average quantity of sewage of each of the ten major sewer districts (see Fig. 1) was estimated as shown in Table 2. The unit quantities used in computing these sewage flows are as follows:

	Unit rates of average sewage flow
Gallons per capita per 24 hr; domestic	70
Gallons per Acre per 24 Hr:	
Industrial	
North Buffalo	10,000
East Buffalo	12,000
South Buffalo	10,000
Commercial	20,000
Infiltration	
North Buffalo	1,000
East Buffalo	2,000
South Buffalo	4,000

A summary of the estimated average sewage quantities, in million gallons daily, is as follows:

Description	1930	1985
Domestic	40.1	77.0
Industrial	31.6	52.0
Commercial	16.0	24.0
Infiltration	34.4	52.2
Totals	122.1	205.2
Totals, in gallons per capita per day	213	186

For the design of sewers and structures serving only part of the city, the estimated average sewage quantities were increased by factors to allow for a different distribution of the future growth than that estimated. These factors varied with the size of the area served and with the estimated density of population, having a maximum value of 1.5. The factors were applied to the domestic, industrial, and commercial quantities, but not to the infiltration.

For area that will overflow to the Buffalo River the intercepting sewer capacities were based on 3.0 times the estimated 1985 average flow, and for all other areas on 2.5 times the estimated 1985 average flow.

Based on rainfall records and a runoff coefficient of 60%, a computation was made to indicate approximately the number and duration of overflows of combined sewage into the waterways. No very exact computation was attempted, but the approximate results are given in Table 6. A proportion of the total sewage flow varying from about one third of one per cent, in the near future, up to almost one and one third per cent in 1985 will be overflowed during rain storms. Kutter's formula, with $n = 0.013$, was used in determining the capacity of all the intercepting sewers.

Careful studies were made to determine the economical balance between the size and slope of the intercepting sewer and pumping costs. In order to utilize all the head available and at the same time keep the sewer below existing and future utilities, much of the main intercepting sewer was built at a flatter grade than the slope of the hydraulic gradient at maximum flow, so that at times of storm the 11-ft and 9-ft portions of the sewer will operate with the hydraulic gradient as much as 2.5 ft above the crown of the sewer.

TABLE 6.—ESTIMATED FREQUENCY AND DURATION OF OVERFLOW FROM INTERCEPTING SEWERS FOR THE SIX SUMMER MONTHS, MAY TO OCTOBER

Intercepting sewer capacity, in gallons per capita per day	Population density per acre	AVERAGE DRY-WEATHER FLOW; 150 GALLONS PER CAPITA PER DAY		AVERAGE DRY-WEATHER FLOW; 200 GALLONS PER CAPITA PER DAY		AVERAGE DRY-WEATHER FLOW; 250 GALLONS PER CAPITA PER DAY	
		Number of overflows	Hours of overflow	Number of overflows	Hours of overflow	Number of overflows	Hours of overflow
400	20	95	230	104	257	....	....
	30	83	187	90	211	98	245
	40	76	161	82	180	90	211
	50	69	135	76	161	83	187
	60	63	117	70	140	79	168
600	20	79	168	82	181	86	195
	30	66	127	70	140	74	153
	40	56	101	61	112	65	124
	50	49	83	53	93	57	103
	60	42	71	47	79	51	89
800	20	68	132	70	140	73	147
	30	54	95	56	101	59	107
	40	44	75	47	79	49	85
	50	37	58	40	64	42	69
	60	32	47	34	52	37	57
1,000	20	59	106	61	111	63	116
	30	45	74	47	59	49	83
	40	36	56	37	60	39	63
	50	29	43	31	47	33	50
	60	24	32	26	35	28	39

A condensed profile of the main intercepting sewer is shown in Fig. 3, and typical sections of the intercepting sewer are shown in Fig. 4. As shown on the profile, the capacity of the sewer varies from 367 to 1,047 gcd, with the higher capacities along the Buffalo River.

For sewers between 3.5 ft and 7 ft in diameter, bidders were allowed to choose any one of four sections: Monolithic concrete, reinforced concrete pipe, brick, and segmental tile. Reinforced concrete pipe (types A and H, Fig. 4) was the choice of bidders in every case, except for monolithic concrete (type B) for earth tunnel in division 0, Fig. 3.

For rock tunnels the horseshoe shape (type C) was used, with a plain concrete lining having a minimum thickness of 8 in. For the larger sewers in open cut, the semi-elliptical section was chosen after extensive investigations of the relative economy of several sections. Drawings of types D, E, and G in

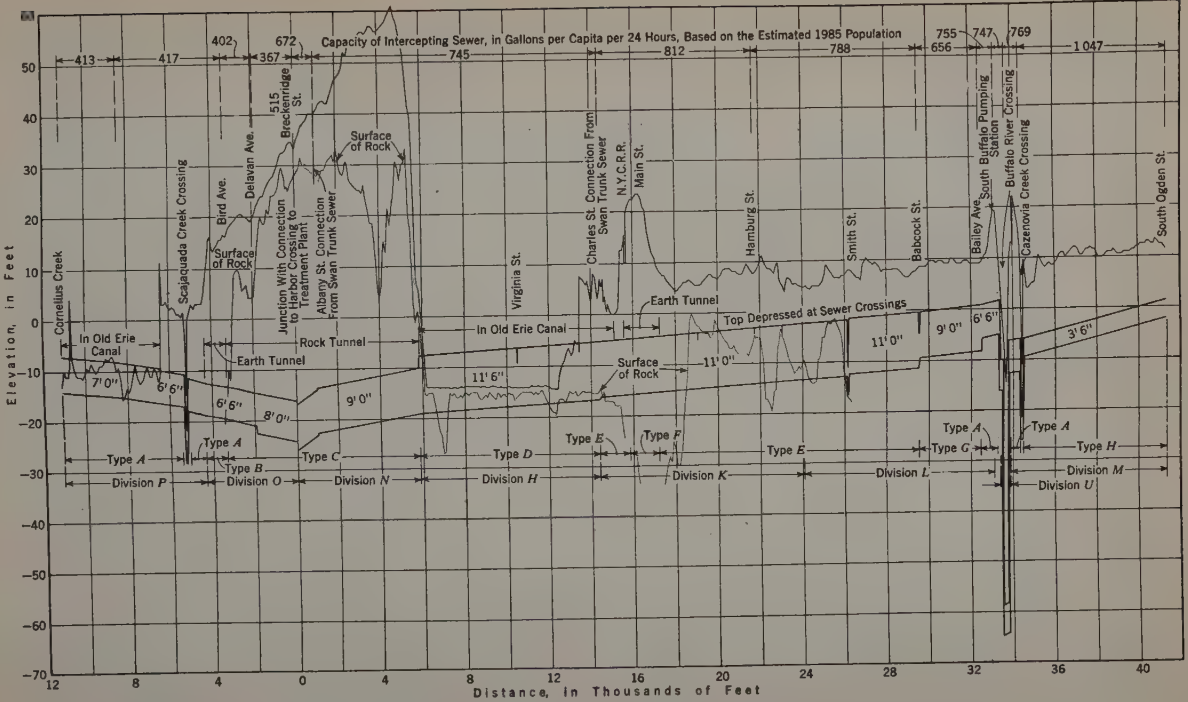


FIG. 3.—GENERAL PROFILE OF INTERCEPTING SEWERS

Fig. 4 show the sizes and dimensions used. "Standard" sections were used for shallow cuts where no extraordinary loading was anticipated. "Heavy" sections were used for deep cuts or where other heavy loads were anticipated, such as from railroad tracks. The entire length of the 11.5-ft, type D section in the abandoned Erie Canal was designed for heavy loads to provide for a proposed relocation of main-line tracks of the New York Central Railroad.

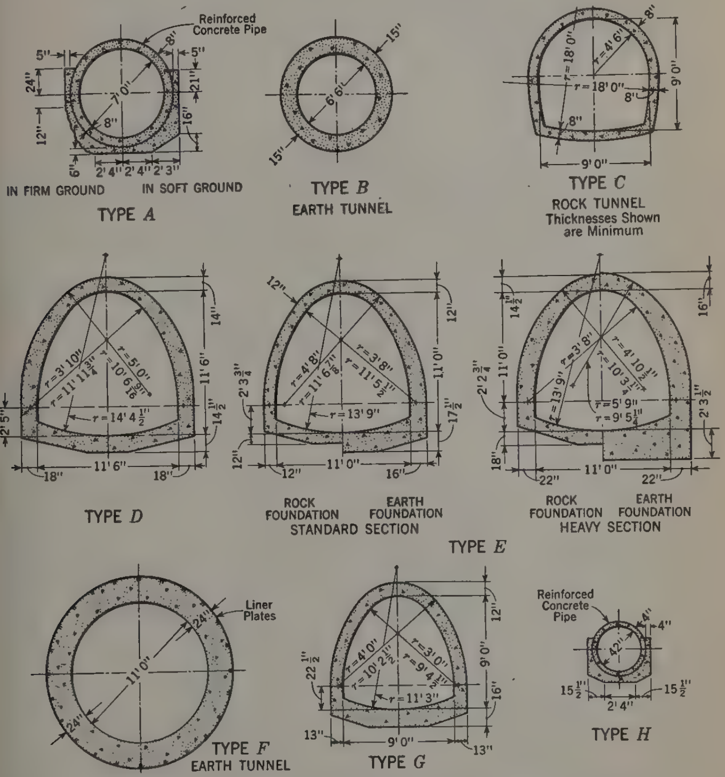


Fig. 4.—Typical Sewer Sections
(All Monolithic Sections Except Type C Contain Steel Reinforcement)

Many special structures were required. Near Bailey Avenue, in the southeastern part of the city, is the South Buffalo pumping station, equipped with four vertical centrifugal pumps having a total rated capacity of 100 mgd. A tunnel crossing under the Buffalo River leads to this pumping station. It comprises two conduits, one 36 in. in diameter and the other, a rectangular section, 4 ft by 6.5 ft. The main crossing under Black Rock Harbor to the

Sewage Treatment Plant comprises two conduits, each 8 ft in diameter. Some 55 intercepting and regulating chambers were built, as well as a large number of special structures including creek and sewer crossings, junction chambers, and several lateral separate sewers.

The estimated final construction cost of the intercepting sewers (exclusive of unsettled claims) may be summarized as follows:

Main intercepting sewer	$5,254,000
Branch sewers	516,000
South Buffalo pumping station	223,000
Harbor crossing	414,000
Total	$6,407,000

The cost of the Main Intercepting Sewer is made up of $4,703,000 for the sewer line proper, including manholes, sheeting left in place, rock excavation, paving, handling utilities, etc.; and $551,000 for special structures, such as intercepting chambers, junction chambers, and river and creek crossings. The sum of $4,703,000 is itemized, as shown in Table 7.

TABLE 7.—MAIN INTERCEPTING SEWER; OVER-ALL COST OF SEWERS, INCLUDING MANHOLES, SHEETING, ROCK, PAVEMENT, UTILITIES, ETC., BUT NOT INCLUDING SPECIAL STRUCTURES

Contract Division (see Fig. 3) (1)	Type (see Fig. 4) (2)	Size, in feet (3)	Length, in linear feet (4)	Cost, in dollars, per linear foot (5)	Contract Division (see Fig. 3) (1)	Type (see Fig. 4) (2)	Size, in feet (3)	Length, in linear feet (4)	Cost, in dollars, per linear foot (5)
P	A	6.5	2,059	51.70	K	E*	11.0	5,074	145.30
P	A	7.0	4,578	41.60	K	E†	11.0	1,273	148.75
O	B	6.5	1,049	132.10	K	E‡	11.0	1,176	120.40
O	C	6.5	1,407	93.00	K	E§	11.0	490	252.20
O	C	8.0	1,877	104.20	K	F	11.0	1,410	260.00
O	C	11.0	504	147.10	L	E*	11.0	1,989	92.70
N	C	9.0	5,720	97.90	L	E†	11.0	2,864	73.60
H	D	11.5	8,582	80.50	L	E‡	11.0	105	105.80
M	A	6.5	495	58.60	L	E§	11.0	85	135.90
M	H	3.5	9,680	28.90	L	F	11.0	370	136.60
....	....	....	....	....	L	G	9.0	2,909	69.50
....	....	....	....	....	L	A	6.5	1,375	57.90

* Standard section, rock foundation. † Standard section, earth foundation. ‡ Heavy section, rock foundation. § Heavy section, earth foundation.

MAIN PUMPING STATION AND SEWAGE TREATMENT PLANT

By special permission of the City of Buffalo, the State of New York, and the United States, the sewage treatment plant is situated on Bird Island, along the west boundary of the city and somewhat north of the east and west center line. The major bases of design are as follows:

Bases of Design

1. Population:

1935	600,000
1939	640,000
1950	750,000

2. Sewage Quantities (All in Million Gallons Daily):

Capacity of intercepting sewer (1985)	563
Capacity of main pumping station	570
1950 basis (population 750,000)	
Average	150
Maximum:	
Clarification and disinfection units	570
Conduits	570
4-hr maximum	570
24-hr maximum	450
2-day maximum	375
3-day maximum	300
Maximum month	200
Minimum (for less than one hour)	80
Near future basis (population 640 000):	
Average	128
Minimum (for less than one hour)	65

3. Sewage Characteristics:

Suspended Solids in Raw Sewage Average (1950):	
Parts per million	180
Pounds per capita per 24 hr	0.3

Suspended Solids Removed and To Be Disposed of:

Period	Percentage removal	Pounds per 24 hr
Average	48.0	108,000
Maximum		
24 hr	38.4	448,000
2 days	38.4	351,000
3 days	38.4	278,000
Month	40.0	135,000

(Assume that, as a yearly average, the volatile matter is 70% of the total suspended solids and that at times of maximum flows it is 50%.)

Quantities of Grit:

Period	Cubic yards per million gallons	Cubic yards per 24 hr
Average	0.15	22.5
Maximum		
24 hr	1.0	200.0
Month	0.25	50.0

4. Grit Chambers:

Basis of design, in million gallons daily	570
Number of units or channels	12
Average depth of sewage, in feet	8.0
Length, in feet	65
Width, in feet	10
Velocity, in feet per second:	
Maximum	1.05
Minimum	0.75

5. Screens:
   - Coarse trash racks with bar-spacing of 5.5 in.
   - Bar screens mechanically cleaned with clear spacing of bars at 0.75 in.

6. Sedimentation Tanks:

Displacement period at average flow of:

Hours	1.33
Minutes	80.00

7. Raw Sludge Quantities:

Percentage of moisture in sludge	96

Other quantities:

Period	Gallons of wet sludge per 24 hr	Pounds of dry solids per 24 hr
Average	324,000	108,000
Maximum		
24 hr	1,345,000	448,000
2 days	1,053,000	351,000
3 days	833,000	278,000
Maximum month	405,000	135,000

(Assumed weight of wet sludge, 62.5 lb per cu ft)

Chemicals for conditioning raw sludge, in pounds per 100 lb of dry solids:

	Average	Maximum
Ferric chloride (100%)	2.0	3.0
Lime (CaO)	8.0	12.0

A general plan of the sewage treatment plant is shown in Fig. 5 and a profile showing the high-water and low-water hydraulic grade lines is shown in Fig. 6. The elevations in Fig. 6 are referred to Buffalo City Datum, which is 575.89 ft above mean tide at New York, N. Y. (1903 level adjustment, U. S. Army Engineers). The gradients show that,

(*a*) With river level at − 5.7 and with a sewage flow of 570 mgd, 330 mgd will be discharged through the submerged outlet and 240 mgd through the overflow weir;

(*b*) With river level at − 9.7 and with a sewage flow of 570 mgd, 500 mgd will be discharged through the submerged outlet and 70 mgd through the overflow weir;

(*c*) With river level at − 5.7, sewage flows of as much as 225 mgd will be discharged through the submerged outlet, without overflow; and,

(*d*) With river level at − 9.7, sewage flows of as much as 470 mgd will be discharged through the submerged outlet, without overflow.

The treatment plant occupies a somewhat triangular shaped site, having an area of 23.0 acres. At the southerly end is the main pumping station which lifts the sewage about 23 ft at average sewage flow. The sewage then flows by gravity through a screen and grit chamber, venturi meters, aerated channels, four sedimentation tanks, and a submerged outfall to the Niagara River.

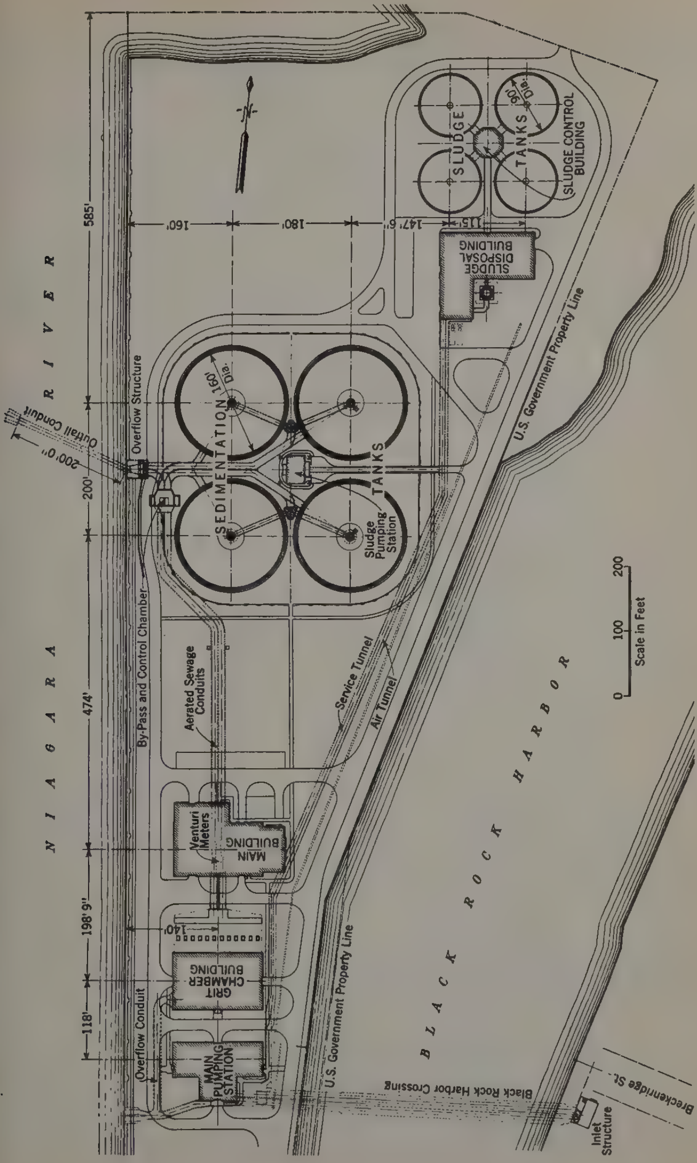


FIG. 5.—GENERAL PLAN OF SEWAGE TREATMENT PLANT

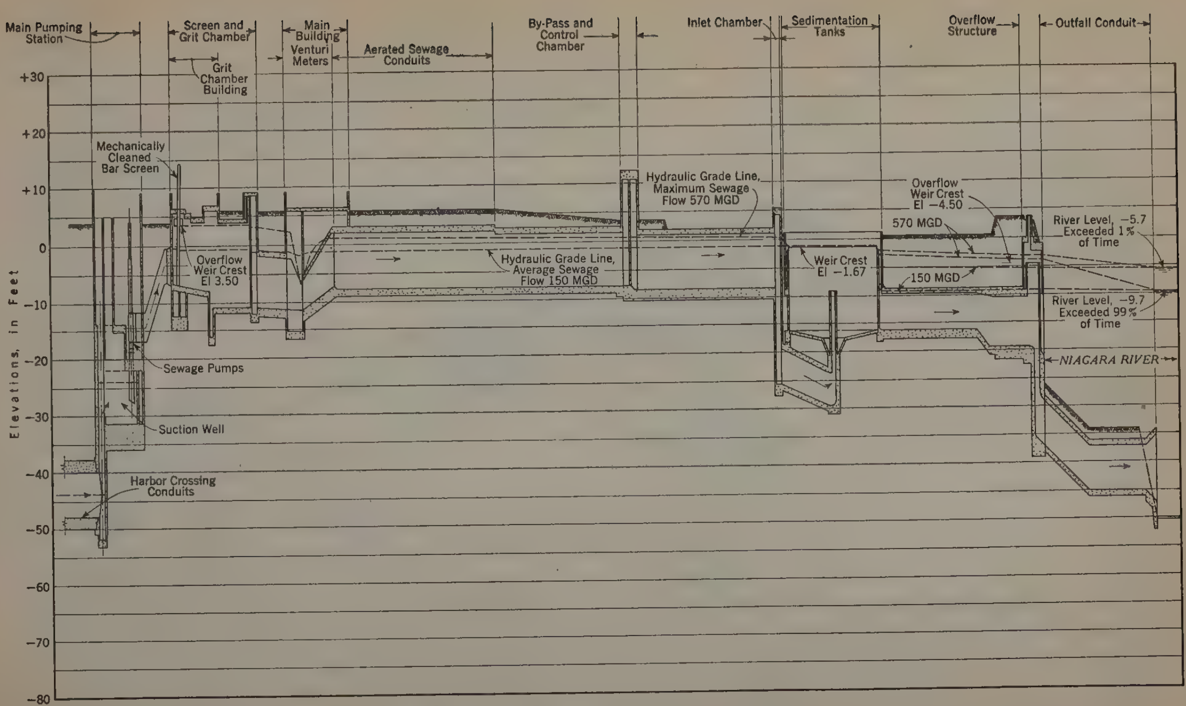


Fig. 6.—Hydraulic Profile, Sewage Treatment Plant

The pumping building contains six vertical, close-coupled, centrifugal pumps, each with a rated capacity of 100 mgd and a low-head capacity of 120 mgd. The total capacity of the pumping station is thus somewhat more than that of the intercepting sewer system. Two of the pumps are fitted with two-speed motors to assist in uniform regulation of flow into the sedimentation tanks.

There are twelve grit chambers. Each pair operates as a unit with one bar screen and pump, and there are no cross connections between these units. Screenings are discharged into grinders and returned thence to the sewage. The grit chambers proper are 60 ft long and 10 ft wide; and they have an average liquid depth of 10 ft.

After leaving the grit chambers, sewage flows through two conduits under the main building, each conduit having a venturi tube. Following the venturi tubes, the two conduits are equipped with plates and piping for aerating the sewage. This section of the conduits is 285 ft long and traverses a part of the site left open, for the present, to provide for future structures. Chlorine is added to the sewage at the far end of these aerated conduits near the sedimentation tanks. The chlorination equipment is housed in the main building.

The sewage then flows through a by-pass and control chamber to four circular sedimentation tanks which provide a displacement period of 80 min at rated capacity. Each of these sedimentation tanks is 160 ft in diameter, and the side-water depth is 14.5 ft. Effluent from the sedimentation tanks flows into the Niagara River about 200 ft from shore, at a depth of about 40 ft.

The suspended solids are pumped from the sedimentation tanks to four sludge storage tanks, having a total capacity of 750,000 cu ft, or 1.0 cu ft per capita. Each tank is 90 ft in diameter, with side and center depths of 27 ft and 38 ft, respectively. Each of the tanks is equipped with a floating cover.

In the center of the space occupied by the sludge storage tanks is a building that houses sludge pumping and control equipment with provision for the delivery of sludge at given rates into one or all of the four tanks, the transfer of sludge from one tank to another, and to the sludge-disposal building (see Fig. 5).

The sludge disposal building contains three de-watering filters of the vacuum type, three flash-type sludge incinerators, and the boiler plant for heating the sludge storage tanks and all buildings. Each drum filter is 11.5 ft in diameter and 14 ft long, with a filtering area of 500 sq ft. It is estimated that these filters will de-water the sludge to a moisture content of 70%, at the average rate of 5.0 lb of dry solids per sq ft per hr when conditioned with about 2.0% ferric chloride and 8.0% lime (expressed as CaO).

Sludge cake from the filters is discharged on to a belt conveyer extending the length of the building which, in turn, discharges to a cross conveyer where the sludge is weighed and elevated to another longitudinal conveyer which delivers it to the sludge incinerator units.

Sludge cake, mixed with previously dried sludge, is delivered to flash dryers in the sludge incinerators; thence it is blown into the furnaces for combustion. Any deficiency in heat units is made up by burning oil or gas. The sludge incinerators include ash-removal equipment, fly ash cyclones, and other appurtenances. Each of the three incinerators has a rated capacity of 100 tons of sludge cake (containing 70% moisture) per 24 hr. Two of the incineration units will probably be sufficient to dispose of the average quantity of sludge.

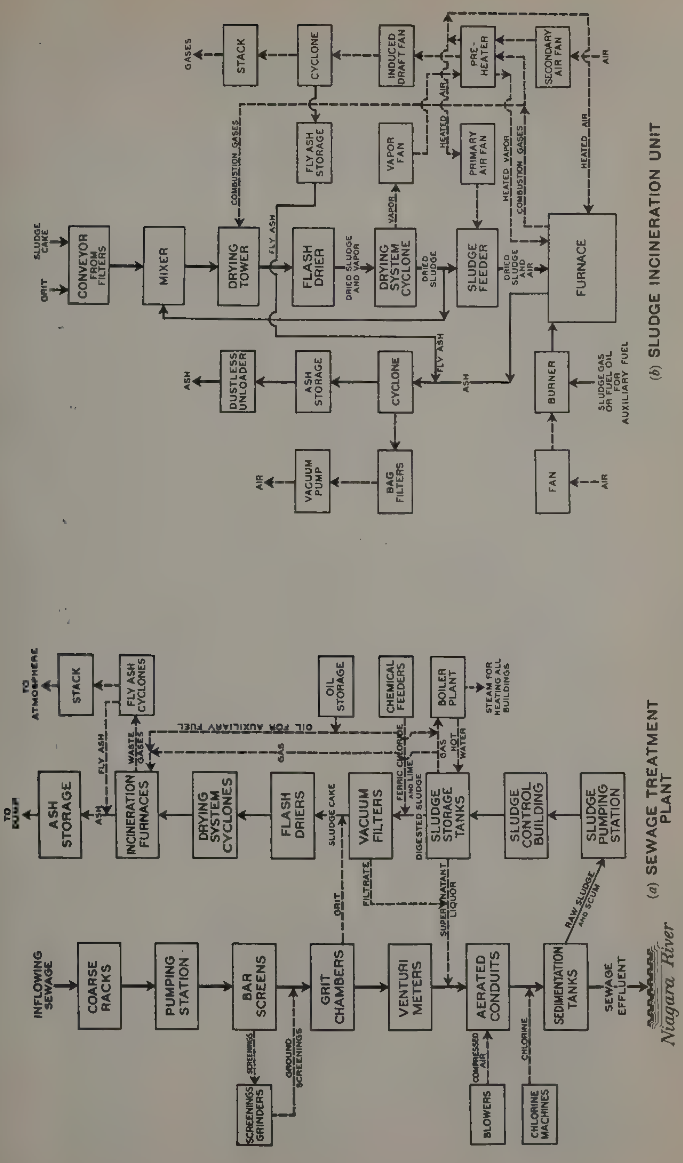
The various features of the sewage treatment plant are connected by a service tunnel along one side of which the main ventilating duct is constructed. The several parts and buildings of the plant where sewage is exposed are provided with large ventilating fans which withdraw air and discharge it into this main ventilating duct. The chimney of the treatment plant, rising 175 ft above the ground, is actually two concentric ducts. The inner of these is for the hot gases from the sludge incinerators and the boiler plant, and the outer, or annular one, is for ventilation and receives the air blown through the main ventilating duct.

The main building houses the office and laboratory, garage, machine shop, blowers for the aerated conduits, chlorine equipment, and the plant water supply, including pumps and elevated tank. A flow diagram of the sewage treatment plant is shown in Fig. 7(*a*), and of a sludge incineration unit in Fig. 7(*b*).

The approximate total cost of the main sewage pumping station and sewage treatment plant was $4,378,891, distributed as follows:

Contract division	Total cost
Preparation of site	$ 166,867
Pumping station and grit chamber	914,429
Sedimentation tanks	772,921
Pumping equipment	324,000
Electrical work	115,537
Sluice gates	49,375
Sludge de-watering equipment	74,965
Incineration equipment	257,500
Sludge disposal building	298,670
Sludge storage tanks	414,807
Brick stack	28,881
Electrical work	35,978
Boiler plant equipment	69,314
Outfall structure	102,897
Main building, service tunnels, sewage conduits, and meters	600,546
Trucks	13,266
Yard water piping equipment	22,327
Roads and walks	54,941
Wall around sedimentation tanks	16,670
Landscaping	20,000
Miscellaneous informal bids	25,000
Total	$4,378,891

The part of this total sum required for sewage treatment is about $3,500,000, which is equivalent to $4.67 per capita or $23,300 per mgd of design capacity.



(a) SEWAGE TREATMENT PLANT

(b) SLUDGE INCINERATION UNIT

FIG. 7.—FLOW DIAGRAMS

A summary of the estimated final costs for the major divisions of the sewage treatment plant construction is shown in Table 8. For the most part, bids were received on a unit price basis. Table 8 shows the high and the low bids of the successful bidders and a weighted average for each kind of construction for which unit price bids were received.

TABLE 8.—SUMMARY OF BIDS ON UNIT PRICE ITEMS, BY SUCCESSFUL BIDDERS; SEWAGE TREATMENT PLANT

Item No.	Description	High	Low	Weighted average
1	Earth excavation, in cubic yards	$ 4.00	$ 1.00	$ 2.90
2	Rock excavation, in cubic yards	15.00	5.00	13.28
3	Selected fill, in cubic yards	3.00	1.00	2.22
4	Embankment, in cubic yards	0.70	0.40	0.512
	Sheeting Left in Place:			
5	Wood, in thousand feet, board measure	60.00	25.00	52.67
6	Steel, in pounds	0.03	0.02	0.0258
	Sheet Piling:			
7	Wood, in thousand feet, board measure	125.00	40.00	93.34
8	Steel, in pounds	0.06	0.04	0.0556
9	Timber piles, in linear feet	1.00	0.35	0.553
10	Steel H-piles, in linear feet	10.00	1.50	2.75
11	Steel reinforcement, in pounds	0.05	0.0425	0.0488
	Concrete, in Cubic Yards:			
12	Class A-1, formed floors and footings	25.00	19.50	20.71
13	Class A-2, formed floors and footings	17.00	13.00	15.48
14	Class B, fill	12.10	10.00	11.38
15	Pre-cast	50.00	35.00	43.95
16	Rubbed finish, in square feet	0.10	0.09	0.0963
	Expansion Joints:			
17	Copper plate, in linear feet	1.10	0.75	0.871
18	Mastic, in square feet	1.00	0.25	0.296
19	Asphalt, in square yards	1.02	0.40	0.785
20	Common brick masonry, in cubic yards	50.00	25.00	34.33
	Cast-Iron Pipe, in Pounds:			
21	Bell-and-spigot, 12 ft	0.057	0.045	0.0486
22	Bell-and-spigot, 3 ft to 12 ft	0.079	0.056	0.0672
23	Bell-and-spigot, specials	0.09	0.076	0.0831
24	Flanged, 12 ft	0.068	0.06	0.0615
25	Flanged, 3 ft to 12 ft	0.10	0.07	0.0925
26	Flanged, specials	0.12	0.10	0.1143
	Structural Metal, in Pounds:			
27	Structural steel	0.08	0.05	0.0570
28	Miscellaneous steel and iron	0.18	0.117	0.156
29	Iron castings	0.163	0.12	0.123
30	Bronze, etc.	1.20	0.52	0.903
31	Aluminum	1.70	1.00	1.36
32	Pipe railing, in linear feet	4.00	2.00	2.03
33	Galvanized floor grating, in square feet	2.50	1.80	1.84

REVENUES AND EXPENSES

Under the act creating it, the Buffalo Sewer Authority has taken over the maintenance and operation of all of the sewers of the city. Thus, estimates of annual operating costs include items for the maintenance of lateral sewers, for sewer cleaning and repairs, and for progressive rehabilitation. In round numbers, the annual budget is summarized as follows:

Item	Annual sum
Administration and expense of main office	$ 80,000
Collecting sewers	223,000
Outlying pumping stations	59,000
Sewage treatment plant	397,000
Construction work, insurance, and miscellaneous	141,000
Debt service	600,000
Total	$1,500,000

The Authority has adopted a two-part rate schedule for raising the annual revenue. This rate schedule provides that 55% of the total sum (or $825,000) shall be derived from a charge based on water consumption; and that the remainder (or $675,000 per yr) shall be derived from a charge to be paid by all taxable property in proportion to the assessed valuation. The portion of the rate applicable to water use is 40% of the water rates, and the portion applicable to assessed valuation is about $0.73 per $1,000.

ACKNOWLEDGMENT

Many individuals have contributed to the progress and outcome of the Buffalo Sewage Disposal Project. Probably the only satisfactory acknowledgment would include a roster of all of the engineers and administrative officers connected with the work. Among those who have devoted considerable effort are the following:

Members of the Buffalo Sewer Authority.—Daniel H. McCarriagher, Chairman, Henry W. Keitzel, Kevin Killeen, William M. Coon, Myron G. Lehman, George W. Lennon, and Christy J. Buscaglia.

Members of the Firm of Greeley and Hansen.—C. R. Velzy (later superintendent of operation for the Sewer Authority), and J. R. Rumsey, Members, Am. Soc. C. E.; F. C. Sellnow and L. G. Williams, Associate Members, Am. Soc. C. E.; and P. E. Langdon, S. M. Clarke, J. K. Geisey, and T. M. Niles. The firm of Greeley and Hansen served as designing and supervising engineers.

The City of Buffalo, N. Y.—Carl Howell, engineer in charge of sewers for the City, Edward P. Lupfer, M. Am. Soc. C. E., directing engineer for the Buffalo Sewer Authority, and G. E. Symons, chief chemist.

U. S. Public Works Administration.—Harold Epstein, project engineer (later general manager in charge of operation and maintenance of the entire sewerage system of the Buffalo Sewer Authority), and E. L. Peterson, supervising engineer.

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PAPERS

AN IMPROVED METHOD FOR ADJUSTING LEVEL AND TRAVERSE SURVEYS

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Synopsis

A simplified method for adjusting level and traverse surveys is presented in this paper. The results are identical with those obtained by least squares, but the method is so simple as to be understandable by any one possessing an elementary knowledge of algebra and surveying. The labor required for this method of adjustment is considerably less than that required by the usual least-squares methods, and is no greater than that of cruder methods.

The paper is divided into three parts: The main body gives detailed instructions for the adjustment, in such form that the computations may be performed by a clerk familiar with the use of a computing machine. For the benefit of those interested, the theory of this method is developed in Appendix I; and Appendix II explains a simple method of solving the adjustment equations.

Introduction

All surveys, whether for the determination of altitudes or of relative horizontal positions, are subject to two general types of error—systematic and accidental. Before any adjustment of a survey is attempted, all known systematic errors should be removed, leaving only those of the accidental type to be disposed of by the adjustment.

Frequent azimuth checks are usually made in a well-conducted traverse survey. A preliminary angle adjustment can be applied, therefore, before the computation of latitudes and departures is undertaken. The angle adjustment is so simple and well understood that it will not be discussed herein.

When latitudes and departures—or differences of latitude and longitude—have been computed for each course in a traverse net, some error will usually appear in the form of imperfect closures in the net. This error is due to two

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general causes—accidental errors in the straight-line measurements (including other errors treated as accidental), and errors of direction that still remain after the preliminary adjustment of the angles. Because of the complex interrelationship of these errors, and the practical difficulties involved in their separation, it is customary, except in the most precise surveys, to treat the traverse closure as a single accidental error with two components; namely, northing and easting (or latitude and longitude). In this paper it is assumed that all preliminary computations have been performed and that unadjusted north and east coordinates have been determined for both extremities of each line in the net. The resultant closure errors, in northing and in easting, will be treated as independent items, either of which may be adjusted without regard for the other.

Similarly, in a level net, the error will appear as imperfect closures, or differences of elevation, at the junction points. The method of adjustment to be described is applicable equally to a level or traverse net; but, because the latter presents a more general problem, having two errors to be adjusted, special attention will be given to the adjustment of a traverse net.

ADJUSTMENT OF A TRAVERSE NET

The method of adjustment to be described herein is based on the so-called "Junction Point method" for the adjustment of levels developed by Mr. D. H. Baldwin of the U. S. Geological Survey.[3] The theory of the new method is presented in Appendix I of this paper.

To illustrate the method and its application to traverse adjustment, a somewhat involved traverse net will be adjusted in detail. This net is shown diagrammatically in Fig. 1, with the junctions numbered for identification. The stations whose positions are to be held fixed as a result of earlier adjustment or of a more precise survey are indicated in Fig. 1 and are listed in Table 1 with their fixed positions. The results of the preliminary computations of north and east coordinates are listed in Table 2, the upper entry representing the beginning, and the lower entry the end, of the line indicated in Column (1), in accordance with the direction arrow on the line in Fig. 1. The items in

TABLE 1.—GIVEN FIXED STATIONS AND POSITIONS

Station (see Fig. 1)	COORDINATES	
	North	East
A	248,367.22	126,478.29
B	328,946.18	129,817.26
C	315,406.20	154,926.18
D	319,029.42	240,792.22
E	250,612.16	232,163.26
F	293,936.85	187,391.63

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[3] For a complete description of the Baldwin method see "Topographic Instructions of the United States Geological Survey," by C. H. Birdseye, M. Am. Soc. C. E., *Bulletin No. 788*, U. S. Geological Survey, pp. 156–160.

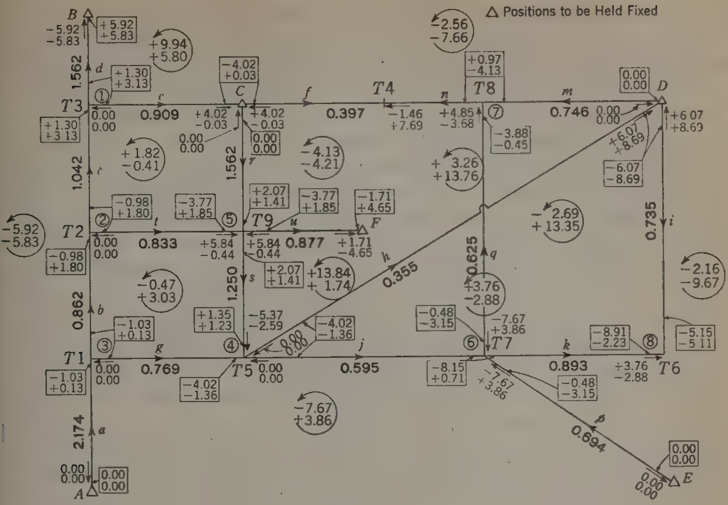

FIG. 1.—ADJUSTMENT DIAGRAM

TABLE 2.—RESULTS OF PRELIMINARY COMPUTATIONS

Line	North	East	COMPUTATION BOOK		Number of stations	Station
			Book	Page		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
a	248,367.22 259,848.50	126,478.29 127,206.14	G20 G20	1 3	23	*A* *T* 1
b	259,848.50 288,642.37	127,206.14 128,889.43	G20 G20	3 7	58	*T* 1 *T* 2
c	288,642.37 312,167.15	128,889.43 127,416.52	G20 G20	7 10	48	*T* 2 *T* 3
d	312,167.15 328,940.26	127,416.52 129,811.43	G20 G20	10 12	32	*T* 3 *B*
e	312,167.15 315,410.22	127,416.52 154,926.15	G20 G20	13 16	55	*T* 3 *C*
f	315,410.22 316,986.28	154,926.15 182,292.37	G20 G20	16 20	56	*C* *T* 4
g	259,848.50 255,027.73	127,206.14 159,716.78	G21 G21	1 5	65	*T* 1 *T* 5
h	255,027.73 319,035.49	159,716.78 240,800.91	G21 G21	5 15	141	*T* 5 *D*
i	319,035.49 261,606.76	240,800.91 239,702.81	G21 G21	15 19	68	*D* *T* 6
j	255,027.73 260,428.60	159,716.78 201,712.94	G22 G22	1 6	84	*T* 5 *T* 7
k	260,428.60 261,610.52	201,712.94 239,699.93	G22 G22	6 9	56	*T* 7 *T* 6
m	319,029.42 318,208.41	240,792.22 217,293.84	G22 G22	10 14	67	*D* *T* 8
n	318,208.41 316,984.82	217,293.84 182,300.06	G22 G22	14 18	70	*T* 8 *T* 4
p	250,612.16 260,420.93	232,163.26 201,716.80	G23 G23	1 5	72	*E* *T* 7
q	260,420.93 318,213.26	201,716.80 217,290.16	G23 G23	5 9	80	*T* 7 *T* 8
r	315,406.20 299,416.02	154,926.18 158,889.16	G24 G24	1 3	32	*C* *T* 9
s	299,416.02 255,022.36	158,889.16 159,714.19	G24 G24	3 6	40	*T* 9 *T* 5
t	288,642.37 299,421.86	128,889.43 158,888.72	G24 G24	7 10	60	*T* 2 *T* 9
u	299,421.86 293,938.56	158,888.72 187,386.98	G24 G24	10 14	57	*T* 9 *F*

Columns (4) and (5), Table 2, are for the computer's reference to the original computation books. Column (7) identifies the stations at the extremities of the various lines, as shown in Fig. 1. The positions shown in Tables 1 and 2 have been computed in plane coordinates. However, this method may be used equally well for adjusting a traverse in geodetic coordinates, differences of latitude and longitude being substituted for differences of northing and easting in the adjustment computations.

Special Conditions.—A few special and unusual conditions are included herein to show how such conditions may be treated in an adjustment. It will be noted that line h in Fig. 1 crosses line q without tying to it. The omission of a junction at this point merely eliminates one equation from the adjustment and presents no special problem. Although lines f and n were surveyed independently in the field, they are in reality a single line in spite of the closure between them, and they are treated as one line in the adjustment, the weight being that of the two lines combined. The same treatment could be applied to lines i and k; but, to illustrate the method, these are treated as separate lines, closing at junction 8 as indicated.

Junction Closures.—A junction closure is the distance by which the computed position of a point on one line differs from the computed position of the same point on another line. It is positive if the computed position on the closing line is the greater, and negative if it is the smaller. Stated otherwise, the junction closure is the distance that must be subtracted algebraically from the position on the closing line to obtain the corresponding position on the line or station closed upon. A junction closure is indicated by a short arrow parallel to the closing line, pointing toward the line closed upon, and by numbers representing the magnitude of the break or closure. The junction closures indicated in Fig. 1 are derived from inspection of Tables 1 and 2. For example, junction 1 is at the end of line c and the beginnings of lines d and e at station T 3. The position of station T 3 is the same on all three lines (see Table 2); therefore, zero closures are indicated between line e and lines c and d. The absence of a closure arrow at the extremity of any line implies a zero closure for that line on the junction involved. At junction 4, station T 5 has the same position on lines g, h, and j; but on line s its position is 5.37 less in northing and 2.59 less in easting. Therefore, line s closes on junction 4 with closures of $-$ 5.37 and $-$ 2.59, as indicated. The position of station C on lines e and f is 4.02 greater in northing and 0.03 less in easting than the fixed position of station C (Tables 1 and 2), as indicated by the closures of these lines on the fixed station. Line r, Fig. 1, starts at station C with the correct final position, and its closure on C, accordingly, is zero. All closures, at junctions and at fixed stations, are determined in the same manner. When dealing with items of northing and easting, it is desirable always to write the northing quantity first, thereby avoiding confusion as well as the necessity of labeling each item.

Circuit Closures.—As the next preliminary step in the adjustment, it is desirable to investigate the circuit closures. Although these do not enter directly into the adjustment, they offer a key to the quality of the field work and preliminary computations, indicating places of apparent weakness and frequently aiding in the discovery of blunders.

The circuit closures may be computed by adding all junction closures counterclockwise around each circuit, changing the sign of a junction closure if its closure arrow is opposed to the counterclockwise route around the circuit. For this purpose, a group of lines connecting two fixed stations is treated the same as a closed circuit. In Fig. 1, the circuit closures are shown within the respective circuits, inside a curved arrow which indicates the direction of closure. Each circuit closure, and all the junction closures contained in it, may be checked from Tables 1 and 2 using Fig. 1 as a guide. Proceeding counterclockwise around the circuit as before, subtract the position of the beginning, and add the position of the end, of each line whose direction arrow agrees with the direction of progress around the circuit; reverse the operations, adding the beginning and subtracting the end, for each line whose direction arrow is against the direction of progress around the circuit. If a circuit connects two fixed stations, assume an imaginary line joining them, and treat the fixed positions as the extremities of this imaginary line. The circuit closures obtained by this method should check exactly those determined previously. Any discrepancy might be due to a mistake in the determination of the circuit closure by either method, or to a mistake in one or more junction closures, or to a combination of these mistakes.

The magnitude of any circuit closure in relation to the length of its circuit is a measure of the precision of the survey around that circuit. An excessively large closure might clearly indicate the existence of a gross blunder and lead to its detection. Large closures of opposite signs in two adjoining circuits indicate weakness, or possibly a blunder, in the line common to these circuits. In general, the algebraic difference between the circuit closures on opposite sides of any line may be used to form a rough preliminary estimate of the correction that the adjustment may be expected to force into that line. Thus, any line that appears to be below the standard of the remainder of the work may reasonably be omitted from the adjustment, in order that its weakness shall not detract from the strength of the remainder of the survey.

Weights.—In the adjustment of a level net, the lines are weighted inversely as their relative lengths. For a traverse, however, it is believed that the number of transit stations in a line offers a better index of the strength of that line. For convenience in solving the adjustment equations, to avoid coefficients either too large or too small, it is desirable that the magnitude of the diagonal terms of the equations be between 1 and 10. This condition can be produced most readily by using weights for the lines, such that their average value will be about 1. Therefore, the effective length of, or number of stations in, each line should be divided by the approximate average of all the lengths, or numbers of stations, before weighting. In this adjustment the lines will be weighted in inverse ratio to the number of stations in each, and the weight of each line in this example will be taken as 50 times the reciprocal of the number of stations in it. The number of stations in each line is listed in Table 2, and the weights are marked on the lines in Fig. 1.

As the preliminary positions and the closures have been computed to two decimal places, it is desirable that the weights be used to one place more, or three places, in order that there shall be no loss of accuracy due to uncertainty in the last decimal figure. If any line has been surveyed with greater or less care than the others, to entitle it to special consideration, its weight factor may be changed accordingly. In this example all lines are considered of equal strength.

TABLE 3.—ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	N	E	S
(1) *ab*		−3.513	+1.042							+ 5.593	+9.134	+12.256
c												
(2) *a*			−2.737	+0.862		+0.833				− 4.865	+0.367	− 4.498
b												
c												
(3) *a*				−3.805	+0.769					0.000	0.000	− 2.174
b												
c												
(4) *a*					−2.969	+1.250	+0.595			− 8.867	−6.322	−15.544
b												
c												
(5) *a*						−4.522				+15.199	+6.563	+19.323
b												
c												
(6) *a*							−2.807	+0.625	+ 0.893	−16.506	+9.963	− 7.237
b												
c												
(7) *a*								−1.768		+ 6.809	−7.754	− 2.088
b												
c												
(8) *a*									− 1.628	− 1.104	−8.959	−10.798
b												
c												
										− 3.741 (−0.749)	+2.992	

Adjustment Form.—After the preliminary examination and preparation are completed, the adjustment may be begun. All junctions between lines of the survey should be numbered, except where lines meet at a fixed station. Thus, there should be a number for each junction for which a correction is to be determined. The form for the table of equations (Table 3) should be prepared, allowing one column for each numbered junction, two absolute columns, and an additional column on each side of the table for later use. Four lines should be allowed for each equation except the first, which requires only three. These groups of lines should be numbered in the same order as the column headings, and the first three lines in each group should be designated *a*, *b*, and *c*, respectively. The fourth line is left blank as a separation between the equations.

In the first equation lines *a* and *b* are combined. As a further guide, a horizontal line may be ruled above each *b*-line.

Equations.—The adjustment equations may now be formed by inspection from Fig. 1. For this purpose, as well as for the solution, a computing machine is almost indispensable. The original equations are written on the *a*-lines, the other lines being reserved for their solution. The diagonal term for each equation is entered in the line and column bearing the number of the corresponding junction, and is always negative. The coefficient of each of the remaining terms of an equation is entered to the right of the diagonal terms, on the line and in the column corresponding to the extremities of the corresponding line in the survey, as will be illustrated.

TABLE 4.—SOLUTION OF ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	N	E	S
(1) *ab*	0.284657	−3.513	+1.042							+ 5.593	+9.134	+12.256
c		(−1.0)	+0.2966							+ 1.5921	+2.6001	+ 3.4888
(2) *a*			−2.737	+0.862		+0.833				− 4.865	+0.367	− 4.498
b	0.411862		−2.428	+0.862		+0.833				− 3.206	+3.076	− 0.863
c			(−1.0)	+0.3550		+0.3431				− 1.3204	+1.2669	− 0.3554
(3) *a*				−3.805	+0.769					0.000	0.000	− 2.174
b	0.285796			−3.499	+0.769	+0.296				− 1.138	+1.092	− 2.480
c				(−1.0)	+0.2198	+0.0846				− 0.3252	+0.3121	− 0.7088
(4) *a*					−2.969	+1.250	+0.595			− 8.867	−6.322	−15.544
b	0.357143				−2.800	+1.315	+0.595			− 9.117	−6.082	−16.089
c					(−1.0)	+0.4696	+0.2125			− 3.2561	−2.1721	− 5.7461
(5) *a*						−4.522				+15.199	+6.563	+19.323
b	0.278242					−3.594	+0.279			+ 9.721	+4.855	+11.262
c						(−1.0)	+0.0776			+ 2.7048	+1.3509	+ 3.1336
(6) *a*							−2.807	+0.625	+ 0.893	−16.506	+9.963	− 7.237
b	0.376081						−2.659	+0.625	+ 0.893	−17.689	+9.047	− 9.782
c							(−1.0)	+0.2351	+ 0.3358	− 6.6525	+3.4024	− 3.6788
(7) *a*								−1.768		+ 6.809	−7.754	− 2.088
b	0.616903							−1.621	+0.210	+ 2.650	−5.627	− 4.388
c								(−1.0)	+0.1295	+ 1.6348	−3.4713	− 2.7070
(8) *a*									− 1.628	− 1.104	−8.959	−10.798
b	0.768640								− 1.301	− 6.701	−6.650	−14.651
c									(−1.0)	− 5.1507	−5.1115	−11.2613
Back solution												
										− 3.741	+2.992	
CN		+1.3026	−0.9761	−1.0326	−4.0159	+2.0720	−8.1546	+0.9678	− 5.1507	(−0.749)		
CE		+3.1328	+1.7963	+0.1322	−1.3599	+1.4063	+0.7142	−4.1332	− 5.1115			
CS		+3.4355	−0.1796	−1.9004	−6.3755	+2.4787	−8.4396	−4.1653	−11.2613			

Diagonal Terms.—The diagonal term for any equation is the negative sum of the weights of all lines entering the correspondingly numbered junction. For example, at junction 1, Fig. 1, it is − (1.042 + 1.562 + 0.909) = − 3.513, as entered in Column (1), line 1*a*, Table 3. For junction 2, it is − (0.862

+ 1.042 + 0.833) = − 2.737, as entered in Column (2), line 2*a*. Similarly, all diagonal terms are determined from Fig. 1, and entered in Table 3, in the *a*-line and column corresponding to the number of the junction involved.

Coefficients.—The remaining coefficients in Equation (1), Table 3, are the weights of the individual lines entering junction 1 from the other numbered junctions. For example, line *c* enters junction 1 from junction 2, and + 1.042 is entered in Column (2), line 1*a*; lines *d* and *e* enter junction 1 from fixed stations and have no coefficients in the table. Similarly, line *b* enters junction 2 from junction 3, and + 0.862 is entered in Column (3), line 2*a*; line *t* enters junction 2 from junction 5, and + 0.833 is entered in Column (5), line 2*a*; line *c* enters junction 2 from junction 1, and + 1.042 is found to be already entered in Column (2), line 1*a*. It is not entered in Column (1), line 2*a*, because that space is to the left of the diagonal term, and only those spaces to the right of the diagonal terms are used. In like manner, the coefficients for all the equations are entered in Table 3.

Absolute Terms.—The absolute terms for northing and easting of each equation may now be computed and entered in Columns N and E, respectively. Each of these terms is the sum of the weighted closures of all lines entering its junction; it is the sum of the products of the weight of each line entering the junction by the sum of the closures in that line computed from the contiguous junction. It is essential that the sign and direction of each closure be observed carefully, the closure sign being held if its direction is toward the junction whose equation is being determined, and reversed if its direction is from the junction. For example, N for Equation (1) is 1.042 (0) + 1.562 (+ 5.92) + 0.909 (− 4.02) = + 5.593; E is 1.042 (0) + 1.562 (+ 5.83) + 0.909 (+ 0.03) = + 9.134. Similarly, for Equation (7), N = 0.625 (+ 7.67 + 4.85) + 0.397 (− 4.02 + 1.46) + 0.746 (0) = + 6.809; E = 0.625 (− 3.86 − 3.68) + 0.397 (+ 0.03 − 7.69) + 0.746 (0) = − 7.754. The absolute terms should be computed to the same number of decimal places as the coefficients.

When all the absolute terms have been computed and entered in Table 3, the adjustment equations are complete and may be checked. The absolute terms should be checked first, as follows: Compute the weighted closures of all lines into all the fixed stations in the same manner in which the absolute terms were computed; write the sum of all these northing closures (− 3.741) at the bottom of the (N)-column and the sum of all these easting closures (+ 2.992) at the bottom of the (E)-column. Including these items, the sum of all entries in each of these columns should be zero, or very nearly so. A small discrepancy in the last place may be due to the rounding off of residual decimals beyond the last significant figure. If preferred, the (N)- and (E)-columns may be checked simultaneously by recording only the sum of the northing and easting closures into the fixed stations (− 0.749), and adding this to both columns to equal zero.

In computing all sums of products, the individual products need not be recorded, but should be accumulated in the computing machine. For this purpose the decimal points should be placed properly on all the dials, and should be observed carefully in all operations. Only the keyboard and multiplier dial are cleared after each multiplication, the product being left in the machine. Negative products are obtained with the reverse multiplier. The product dial is cleared only when the desired sum has been recorded.

(S)-Column.—The (S)-column at the right of Table 3 offers a further check on the formation of the equations, as well as a step-by-step check on the solution. The (S)-item for each equation is determined by adding to the negative sum of the weights of all lines entering a junction from fixed stations the entries in the (N)-column and (E)-column. For example, at junction 1, lines *d* and *e* enter from fixed stations; therefore S1 equals minus the sum of the weights of these lines plus N plus E, or $-(1.562 + 0.909) + 5.593 + 9.134 = +12.256$. At junction 3, only line *a* enters from a fixed station; hence $S3 = -2.174 + 0.000 + 0.000 = -2.174$. There are no fixed stations contiguous to junction 2; therefore, S2 equals the sum of N and E, or $-4.865 + 0.367 = -4.498$.

When the (S)-column is completed, the sum of all entries on line 1*a* through N and E should equal S1 exactly. The sum of all entries in Column (2) and on line 2*a* through N and E should equal S2. Similarly, the sum of Column (3) and line 3*a* should equal S3. Thus, each equation is checked by adding its column down to the diagonal term, then across through E, to equal its (S)-entry. A mistake in any coefficient will appear as a discrepancy in the check of the two equations containing that term. A discrepancy peculiar to a particular equation indicates a mistake in either its diagonal term or its (S)-term.

Solution of Equations.—The equations in Table 3 may now be solved by the Doolittle method[4] as shown in Table 4. The detailed solution especially adapted and simplified for use with a computing machine is explained with complete instructions in Appendix II. The (N)-, (E)-, and (S)-columns are treated identically in the forward solution, the check on the (S)-column being applied in the usual manner after each step. In the back solution, an independent computation is made for CN, CE, and CS for each equation, using only the appropriate items for each, and omitting the other two quantities from each computation. The back solution is checked, step by step, by adding CN, CE, and -1.0000 for each equation; the sum should equal CS for that equation, very nearly.

Corrections.—The (CN)- and (CE)-items of the back solution are the required corrections to be applied to the corresponding junctions of Fig. 1. These are rounded off to two decimal places and are shown in small rectangles with arrows pointing to the line at the extremity of which the correction is applied. It should be borne in mind that these corrections apply to the position closed upon at each junction. The correction to the extremity of the closing line may be determined by subtracting the junction closure from the computed correction. Thus, at junction 1, the computed correction applies equally to lines *c*, *d*, and *e*. At junction 7, the computed correction applies to lines *m* and *n*; the northing correction to the end of line *q* is + 0.97 − (+ 4.85) = − 3.88; its easting correction is − 4.13 − (− 3.68) = − 0.45. The corrections to lines at fixed stations are equal and opposite to the closures on these stations. The difference between the corrections at the ends of a line represents the correction to be distributed or prorated through the line.

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[4] "Adjustment of Observations," by T. W. Wright and J. F. Hayford, Second Edition, pp. 114–121. Detailed instructions for the solution are given in several publications of the U. S. Coast and Geodetic Survey.

Final Test.—At this point the adjustment of the junctions is complete, corrections having been determined for each extremity of each line. One final check may be applied, however, to test the efficacy of the adjustment. According to the theory of least squares, the most probable values for the corrections should satisfy the condition that the sum of the weighted corrections applied to all lines around a given junction will be zero. This test may be applied around each numbered junction by multiplying the weight of each line entering it by the difference of the corrections applied to its extremities. For this purpose, the N- and E-corrections may be tested together, the corrections at the far end of each line being applied with their proper signs, and those at the near end with reversed signs. For example, at junction 1, 1.042 (− 0.98 + 1.80 − 1.30 − 3.13) + 1.562 (+ 5.92 + 5.83 − 1.30 − 3.13) + 0.909 (− 4.02 + 0.03 − 1.30 − 3.13) = 0.01. In view of the number and magnitude of the items included in this test, the discrepancy of 0.01 is reasonable, and indicates a satisfactory adjustment. The same test may be applied to all numbered junctions in the net. In using lines *f* and *n* to check at junction 7, the break at station *T* 4 should be considered, thus: 0.397 (− 4.02 + 0.03 + 1.46 − 7.69 − 0.97 + 4.13) + 0.746 (0.00 + 0.00 − 0.97 + 4.13) + 0.625 (− 0.48 − 3.15 + 3.88 + 0.45) = 0.00. In this example, the discrepancies at the eight numbered junctions vary from 0 to 0.02, with an average value of 0.007.

SUMMARY

A brief summary of the operations required for a complete adjustment may be of value in the formation of a mental picture of the entire process. For a traverse net, the preliminary computations should be completed and the N- and E-coordinates, or the latitude and longitude, should be determined for each junction. For a level net the orthometric elevation of each junction point should be determined. Then the operations of the adjustment should be performed in the following order:

1.—Prepare a diagram of the survey, showing junction closures, circuit closures, and weights of all lines. Number all junctions, except those at fixed stations.

2.—Prepare a form similar to Table 3 for the adjustment equations.

3.—Compute the diagonal terms as the negative sum of the weights of all lines entering each numbered junction.

4.—Enter the weight of each line in the space numbered according to the junction numbers at the ends of that line, using only those spaces to the right of the diagonal terms.

5.—Compute and enter in the (N)- and (E)-columns, respectively, the weighted closures in northing and easting of all lines entering each numbered junction. In a level adjustment, of course, there would be only one absolute column.

6.—Check the (N)- and (E)-columns by summing them to zero with the weighted closures of all lines entering all fixed stations.

7.—Compute the (S)-entry for each equation as the negative sum of the weights of all lines entering its junction from fixed stations, plus its (N)- and (E)-entries. Add each equation down its column to its diagonal term, then across through its absolute terms to equal its (S)-term, exactly.

8.—Solve the equations by the Doolittle method, forward and back solutions.

9.—On the adjustment diagram, enter the results of the back solution as corrections to the lines closed upon at the numbered junctions.

10.—Subtract the junction closures from the computed corrections to determine the corrections at the ends of the closing lines.

11.—Test the adjustment by comparing with zero the sum of the weighted corrections in all the lines around each numbered junction.

12.—Distribute or prorate the difference between the end corrections of each line to the stations in that line, to complete the adjustment.

CONCLUSION

It is believed that the results of any survey, even a poor one, can be improved by appropriate adjustment, so that all parts of the survey shall fit together with mathematical nicety after the computations are completed. In other words, any survey worth making is worth improving at small additional cost by careful adjustment. The relatively small cost of adequately adjusting a survey undoubtedly improves the results to a greater degree than could be obtained by a similar expenditure for improved field technique. Obviously, poor work cannot be made good by adjustment, but the data resulting from any survey can be improved in value and usefulness if all mathematical inconsistencies are removed. When the funds available for a survey are limited, it is believed desirable to reserve a portion of the money for the purpose of adjustment.

In traverses, except for the most precise geodetic surveys, it is customary to combine all accidental errors in the closures of latitude and longitude, or latitudes and departures. In this respect, practically all methods of adjustment agree. Beyond this, variations exist, from adjustment by "estimation" or judgment to adjustment by least squares. For any adjustment, regardless of method, it is desirable that a diagram be prepared to show the various closures and the relative weights of the respective lines. The diagram, although not an essential part of the adjustment, is an invaluable aid in the determination and distribution of the adjustment corrections.

It is generally recognized that the method of least squares offers the best adjustment possible in accord with the theory of probability. Although the method described herein is not derived through a consideration of the squares of residuals or errors, it is based on the principle that the correction at any junction shall be a weighted mean of the corrections brought in from the contiguous junctions (see Appendix I). This principle is in perfect harmony with the principle of least squares, and the two methods give identical results.

It may be emphasized, furthermore, that the adjustment equations that are set up by inspection from the adjustment diagram are identical in every respect with the normal equations that would be obtained by the use of the observation-equations method in least squares. This new method has a definite advantage over the latter in that the equations may be written by inspection, requiring no auxiliary tables. It has similar advantages over the condition-equations method, in that a table of correlate equations is not needed, and the solution of the adjustment equations gives the desired corrections directly, without the necessity of applying the correlates to the correlate equations and computing the line corrections before the junction corrections can be determined.

It is believed that the method of adjustment described herein is so simple as to be easily understood by any one familiar with the elements of surveying and of algebra. Although the Doolittle method is suggested as the shortest known method of solving the type of simultaneous equations obtained, these equations may be solved equally well by the usual algebraic methods. (For algebraic solution, the equations should be written in complete form; see Appendix I.) Although the method described is identical with least squares, it is not expected to have the psychological effect that the name "least squares" has upon most surveyors—the suggestion of something too complicated to be understood except by an expert mathematician. Some technical understanding is helpful, of course, in preparing and studying the closure diagram, but the mechanics of the adjustment are such that, if the instructions are observed carefully, a complete adjustment can be made by a clerk familiar with the use of the computing machine.

As compared with cruder methods of adjustment, this method requires little additional work for the theoretically most probable distribution of the errors. The only work peculiar to this method is the writing and solving of the simultaneous equations. In this respect there is a compensating saving of labor, particularly in a traverse adjustment, in that corrections for both north and east coordinates are obtained simultaneously, whereas in most methods these must be adjusted separately. As compared with the Baldwin method for a level adjustment, this method gives a more balanced adjustment, eliminates the need for exercise of judgment in the selection of junctions, and permits adjustment of an entire network at a single set-up. The time required for adjusting an ordinary level net by this method compares favorably with that required by the Baldwin method.

Because this method is suitable for the adjustment of any type of line survey, another possibility becomes apparent. In many surveys it happens that traverse and levels are run over the same routes for the purpose of determining horizontal and vertical control for all monuments. If the lines are so run that the weights to be used for adjusting the levels are in the same ratio as the weights to be used for adjusting the traverse, or can be made so, then both traverse and levels can be adjusted at the same time by using three absolute columns in the adjustment equations instead of one or two. With three absolute columns, the (S)-column should include the effect of all three, and the solution may be completed in exactly the same manner.

ACKNOWLEDGMENTS

This method of adjustment was developed in the Section of Computing, of the U. S. Geological Survey, between 1936 and 1938. The short-cut approach to the theory through simple algebra was suggested by Mr. Norris; Mr. Speert arranged the mechanics of the adjustment in collaboration with other members of the Section. This paper is published by permission of the Director of the Geological Survey, United States Department of the Interior.

APPENDIX I

THEORY

As a necessary preliminary to the development of the new theory, the Baldwin method will be reviewed briefly.[3]

In the simple level net shown in Fig. 2, line *A D C* runs from bench mark *A* (the elevation of which is fixed) to another fixed bench mark, *C*, closing on bench mark *C* with an error of + 0.03 ft as indicated by the small arrow at point *C* and the number + 0.03 (this means that the elevation of bench mark *C*, as determined by line *A D C*, is 0.03 ft greater than the fixed elevation of bench mark *C*). Similarly, line *B D* begins at fixed bench mark *B* and closes on bench mark *D* with an error of − 0.05 ft (or the elevation of bench mark *D* as determined by line *B D* is 0.05 ft less than the unadjusted elevation determined by line *A D C*). Assume also that the lengths of *A D*, *B D*, and *D C* are 5, 10, and 8 miles, respectively.

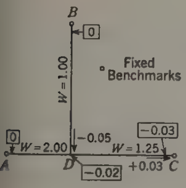

FIG. 2

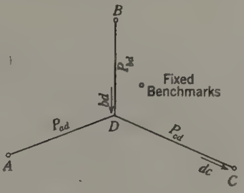

FIG. 3

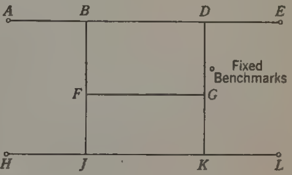

FIG. 4

ADJUSTMENT DIAGRAMS

For the adjustment, the lines are to be weighted inversely as their lengths or in the ratio 2.00 : 1.00 : 1.25, as indicated in Fig. 2. This adjustment is based on the principle that the adjusted elevation of bench mark *D* is to be a weighted mean of the elevations brought in through the various lines tying to it. This principle is in strict accord with the principle of least squares and may be readily derived from it. The weighted mean is most conveniently obtained by considering corrections and closures instead of actual elevations. The correction to line $A\,D$ at point A (as shown in a small rectangle in Fig. 2) is zero; the correction to line $C\,D$ at point C (to correct the closure of + 0.03 ft) must be − 0.03; the correction to line $B\,D$ at point B is zero. As there are no breaks in continuity between bench mark D and the indicated corrections at points A and C, the corrections at these points may be used for weighting at point D; in order to refer to a common elevation at point D, the closure of − 0.05 must be added to the correction at point B before the latter may be weighted at bench mark D. Then the weighted correction at point D may be computed as the sum of the products of the various line corrections by the weights of their respective lines, divided by the sum of the weights; that is,

$$D = \frac{2.00(0) + 1.00(-0.05) + 1.25(-0.03)}{2.00 + 1.00 + 1.25} = -0.02 \text{ ft.}$$

To express this principle in a general equation (see Fig. 3), let A, B, C, and D represent the corrections at points A, B, C, and D, respectively; ad, bd, and cd represent the mathematical breaks or closures in the lines A to D, B to D, and C to D, respectively; and P_{ad}, P_{bd}, and P_{cd} represent the weights of the respective lines. Then $D = \dfrac{P_{ad}\,(A + ad) + P_{bd}\,(B + bd) + P_{cd}\,(C + cd)}{P_{ad} + P_{bd} + P_{cd}}$;

or

$$\begin{aligned}(P_{ad} + P_{bd} + P_{cd})\,D &= P_{ad}\,(A) + P_{bd}\,(B) + P_{cd}\,(C) \\ &\quad + P_{ad}\,(ad) + P_{bd}\,(bd) + P_{cd}\,(cd) \qquad (9)\end{aligned}$$

Transposing all terms to the right side of the equation,

$$\begin{aligned}0 &= -(P_{ad} + P_{bd} + P_{cd})\,D + P_{ad}\,(A) + P_{bd}\,(B) + P_{cd}\,(C) \\ &\quad + P_{ad}\,(ad) + P_{bd}\,(bd) + P_{cd}\,(cd) \qquad (10)\end{aligned}$$

Note that cd represents the closure or breaks in the line C to D, whereas dc represents the same closure or breaks in the reverse direction; thus,

$$cd = -dc \qquad (11)$$

Equation (10) represents a general equation for the correction at any junction in terms of the corrections at other points and the mathematical breaks in continuity of the lines connecting the junction with the other points. Ordinarily, the Baldwin method has been used to determine the correction at a particular junction when the corrections at the other points have been known.

Extension of Baldwin Method.—The same principle may be applied to a more complicated level net, as indicated in Fig. 4. Denote the correction to any junction by the letter used to designate that junction; the weight of a line by the letter P and the subscript letters of the ends of the line; and the mathematical breaks in continuity in any line by the lower-case letters of the ends of the line, thus: The break or closure in B to D is bd, that in D to B is $db = -bd$, etc.

As A, E, H, and L are fixed points, their corrections are known from the closures at these points, and need not be computed. Equations similar to Equation (10) may be written for each of the remaining junctions. Thus, at B,

$$\begin{aligned} 0 = -(P_{ad} + P_{bd} + P_{bf})\,B + P_{ad}\,(A) + P_{bd}\,(D) + P_{bf}\,(F) \\ + P_{ab}\,(ab) + P_{bd}\,(db) + P_{bf}\,(fb) \ldots\ldots\ldots\ldots (12) \end{aligned}$$

If $P_{ab} + P_{bd} + P_{bf}$ (the sum of the weights of all lines entering junction B) is designated P_b, and $P_{ab}\,(ab) + P_{bd}\,(db) + P_{bf}\,(fb)$ (the sum of the products of the breaks of the lines into the junction by the weights of these lines) is designated Q_b, Equation (12) becomes

$$0 = -P_b\,(B) + P_{ab}\,(A) + P_{bd}\,(D) + P_{bf}\,(F) + Q_b \ldots\ldots (13)$$

Similarly, equations for the other junctions to be adjusted may be written as follows: At D,

$$0 = -P_d\,(D) + P_{bd}\,(B) + P_{de}\,(E) + P_{dg}\,(G) + Q_d \ldots\ldots (14)$$

at F,

$$0 = -P_f\,(F) + P_{bf}\,(B) + P_{fg}\,(G) + P_{fj}\,(J) + Q_f \ldots\ldots (15)$$

at G,

$$0 = -P_g\,(G) + P_{gk}\,(K) + P_{fg}\,(F) + P_{dg}\,(D) + Q_g \ldots\ldots (16)$$

at J,

$$0 = -P_j\,(J) + P_{hj}\,(H) + P_{fj}\,(F) + P_{jk}\,(K) + Q_j \ldots\ldots (17)$$

and, at K,

$$0 = -P_k\,(K) + P_{jk}\,(J) + P_{gk}\,(G) + P_{kl}\,(L) + Q_k \ldots\ldots (18)$$

As there are six unknown junction corrections and six equations, a rigid solution is possible.

Equations (13) to (18) may be written most conveniently by inspection in the form of a table, the unknown corrections being used to designate the columns, in which only the numerical coefficients need be placed (see Table 5).

If the equations are written in the same sequence as the columns, it will be seen that the negative terms form a diagonal, about which the table is symmetrical, except for the Q-column; that is, all coefficients to the left (or to the right) of any negative term appear again above (or below) it in the same relative position. Thus, except for the Q-column, the table may be read across or down with the same results. This symmetry makes it possible to condense the table by omitting all coefficients to the left of the "diagonal" terms. Then each equation is read down to its diagonal term and across through its absolute term.

In this form the equations may be solved readily by the Doolittle method[4] to give the required junction corrections (see Appendix II).

TABLE 5.—ADJUSTMENT EQUATIONS

At point:	*B*	*D*	*F*	*G*	*J*	*K*	*Q*
B	$-P_b$	$+P_{bd}$	$+P_{bf}$				$+Q_b$ $+P_{ab}$ (*A*)
D	$+P_{bd}$	$-P_d$		$+P_{dg}$			$+Q_d$ $+P_{de}$ (*E*)
F	$+P_{bf}$		$-P_f$	$+P_{fg}$	$+P_{fj}$		$+Q_f$
G		$+P_{dg}$	$+P_{fg}$	$-P_g$		$+P_{gk}$	$+Q_g$
J			$+P_{fj}$		$-P_j$	$+P_{jk}$	$+Q_j$ $+P_{hj}$ (*H*)
K				$+P_{gk}$	$+P_{jk}$	$-P_k$	$+Q_k$ $+P_{kl}$ (*L*)

This method of adjustment is not only suitable for levels, but it is even more advantageous in its application to a traverse net. Instead of a single closure error at each junction, a traverse survey has two closures—in latitude and in longitude, or in northing and easting. These two closures are considered independently and, in other methods of adjustment, require individual treatment However, by the method presented herein, the equations for the corrections at any particular junction are identical except for the absolute or *Q*-terms. Thus, both sets of equations may be combined in a single table, similar to Table 5, but having two absolute columns—one for northing and one for easting The effort required for the solution of both sets is only slightly greater than that required for one alone.

To provide a check on the formation and solution of the equations, an extra column is added to the table. The items in this column are dimensionless numbers, each being the sum of the coefficients and absolute terms of its equation. This sum column is not an essential part of the adjustment, but makes it possible to check each step before proceeding to the next.

APPENDIX II

SIMPLIFIED SOLUTION OF ADJUSTMENT EQUATIONS BY THE DOOLITTLE METHOD

The Doolittle method is a semi-mechanical method for solving simultaneous equations of the symmetrical type formed in this method of adjustment. It is much quicker than the regular algebraic solution, especially when there are many equations to be solved simultaneously, and is particularly adaptable to the use of a computing machine. By this method, as by others, the equations are combined successively, one of the unknowns being eliminated at each step When all the equations have been combined, the resulting equation contains only one unknown, which may be evaluated readily. In the back solution, the evaluated unknowns are substituted successively back into the combined equations for the evaluation of the preceding unknowns.

In Tables 3 and 4, the original equations have been written on the *a*-lines, and each equation is read down its column to the diagonal term, then across through the absolute terms. The *b*-lines, in Table 4, contain the combined equations, each of which includes the influence of those preceding it. Because of the elimination of one additional unknown, each combined equation contains one term less than the preceding one. The combined equations, on the *b*-lines, are read straight across. The equations on the *c*-lines are mathematically identical with the corresponding *b*-equations, but they express the principal unknown as an explicit function of the remaining unknowns. The items on the *c*-lines are the multipliers to be used in the formation of the combined equations on the succeeding *b*-lines. It is obvious that in the first equation the *a*- and *b*-lines are identical and need be written only once.

The procedure for combining and solving the equations follows a few definite rules; when once mastered, it becomes almost mechanical and requires practically no mental effort. The method is best illustrated by completing the solution of the equations in Table 3. The complete solution is shown in Table 4.

The first step in the solution is the formation of Equation (1*c*) from Equation (1*b*). Equation (1) represents the correction at junction 1 in terms of the corrections at all other junctions; therefore the entry in Column (1) (the diagonal term of Equation (1*a*)) will be called the principal term of Equation (1*b*). Similarly, the *b*-item under each diagonal term will be called the principal term of its equation. To express the correction at junction 1 as an explicit function of the other corrections, its coefficient should be made − 1.0. This is done by multiplying all items on line 1*b* by the negative reciprocal of the principal term. The negative reciprocal (0.284657) of − 3.513 is entered, for convenience, in Column R at the left of the table and is multiplied by the items on line 1*b* to give the items shown on line 1*c*. Line 1*c* may now be checked by adding it across through the absolute columns, including − 1.0 for the first term, to equal the (S)-item, very nearly. A small difference may be due to the dropped residual decimals beyond the last significant figure. In general, the *c*-lines should be recorded to one decimal place more than the *b*-lines, in order to retain the same number of significant figures.

The next step in the solution is the formation of Equation (2*b*) by combining Equation (1*b*) with Equation (2*a*). In this step, everything to the left of Column (2) is ignored and each item in Equation (1*b*) is multiplied by the (1*c*)-term in Column (2); then it is added to the (2*a*)-item in its own column. Thus, Equation (2*b*) is formed as follows:

$1b\text{-}2(+1.042) \times 1c\text{-}2(+0.2966) + 2a\text{-}2(-2.737) = 2b\text{-}2(-2.428)$

$1b\text{-}3(0) \times 1c\text{-}2(+0.2966) + 2a\text{-}3(+0.862) = 2b\text{-}3(+0.862)$

$1b\text{-}5(0) \times 1c\text{-}2(+0.2966) + 2a\text{-}5(+0.833) = 2b\text{-}5(+0.833)$

$1b\text{-N}(+5.593) \times 1c\text{-}2(+0.2966) + 2a\text{-N}(-4.865) = 2b\text{-N}(-3.206)$

$1b\text{-E}(+9.134) \times 1c\text{-}2(+0.2966) + 2a\text{-E}(+0.367) = 2b\text{-E}(+3.076)$

$1b\text{-S}(+12.256) \times 1c\text{-}2(+0.2966) + 2a\text{-S}(-4.498) = 2b\text{-S}(-0.863)$

Each item is designated by its line and column, thus: Item 2*a*-3 is on line 2*a*, Column (3). Equation (2*b*) may now be checked by adding it across to equal its (S)-term, very nearly. Equation (2*c*) is formed from Equation (2*b*) in exactly the same manner as Equation (1*c*) was formed from Equation (1*b*). It is checked by adding it across from − 1.0 through the absolute columns to equal its (S)-term, as before.

Equations (1*b*), (2*b*), and (3*a*) are now combined to form Equation (3*b*). In this step, everything to the left of Column (3) is ignored, and the *c*-items in Column (3) are used as multipliers for their respective *b* equations. As there is no item in 1*c*-3 (giving a zero multiplier), Equation (1*b*) drops out of this step. Each item in Equation (2*b*) is multiplied by item 2*c*-3 and then added to the (3*a*)-item in its own column to form Equation (3*b*).

Equation (3*b*) is added across to check on its (S)-term. Equation (3*c*) is formed from Equation (3*b*), and checked. This same routine is followed throughout the forward solution. The first four equations are all short ones and are combined easily, as explained.

Equation (5*b*) is formed by combining Equations (1*b*), (2*b*), (3*b*), (4*b*), and (5*a*). In this step, everything to the left of Column (5), Table 4, is ignored, and the *c*-items in Column (5) are used as multipliers for their respective *b*-equations. As there is no item in 1*c*-5, Equation (1*b*) drops out of this step. To compute an item in any column on line 5*b*, multiply each *b*-item in the column by the *c*-item of its equation in Column (5), and add the products to the (5*a*)-item above the term being computed, thus:

$$\begin{array}{llllllll}
2b\text{-}5 \times 2c\text{-}5 & + & 3b\text{-}5 \times 3c\text{-}5 & + & 4b\text{-}5 \times 4c\text{-}5 & + & 5a\text{-}5 & = 5b\text{-}5 \\
 & & & & 4b\text{-}6 \times 4c\text{-}5 & & & = 5b\text{-}6 \\
2b\text{-N} \times 2c\text{-}5 & + & 3b\text{-N} \times 3c\text{-}5 & + & 4b\text{-N} \times 4c\text{-}5 & + & 5a\text{-N} & = 5b\text{-N} \\
2b\text{-E} \times 2c\text{-}5 & + & 3b\text{-E} \times 3c\text{-}5 & + & 4b\text{-E} \times 4c\text{-}5 & + & 5a\text{-E} & = 5b\text{-E} \\
2b\text{-S} \times 2c\text{-}5 & + & 3b\text{-S} \times 3c\text{-}5 & + & 4b\text{-S} \times 4c\text{-}5 & + & 5a\text{-S} & = 5b\text{-S}
\end{array}$$

always using the *c*-items in Column (5) as the multipliers for the corresponding *b*-items in the other columns. Equation (5*b*) is now checked by adding across to 5*b*-S. Equation (5*c*) is formed by multiplying Equation (5*b*) by the negative reciprocal of its principal term, and is checked by adding across, including the − 1.0, to equal 5*c*-S, very nearly. Equation (6*b*) is formed by combining the previous *b*-equations with Equation (6*a*) as already explained, and is checked on 6*b*-S.

Back Solution.—The same routine is followed to the formation and check of Equation (8*c*), which expresses the correction at junction 8 as an explicit function of no other unknown. In other words, the corrections at junction 8 appear in the absolute columns on line 8*c*, and are − 5.1507 in northing and − 5.1115 in easting. As the first unknowns evaluated, these are entered on lines CN and CE, respectively, in Column (8) of the back solution. To provide a check, 8c-S is entered in CS-8. Then, in Column (8), CN + CE − 1.0000 should equal CS, very nearly. (This check was already applied on line 8*c*.)

The corrections at junction 7 may be evaluated next. Equation (7*c*) expresses each correction at junction 7 as an explicit function of the corresponding correction at junction 8, which is already known. The corrections at junction 8 (which will be called, collectively, C8) are therefore substituted into Equation (7*c*) to evaluate C7. As these corrections, in reality, are independent variables, it will be necessary, in the back solution, to substitute them separately, and to evaluate but one correction at a time. Thus:

$$\text{CN-8} \times 7c\text{-8} + 7c\text{-N} = \text{CN-7}$$
$$\text{CE-8} \times 7c\text{-8} + 7c\text{-E} = \text{CE-7}$$

and,

$$\text{CS-8} \times 7c\text{-8} + 7c\text{-S} = \text{CS-7}$$

The (CS)-item is not a correction but is used as a check on the others and is computed in the same manner. For the check, as before, CN-7 + CE-7 − 1.0000 = CS-7. Similarly, C6 is computed by substituting C7 and C8 into Equation (6*c*), and is checked by adding CN, CE, and − 1.0000 to equal CS, very nearly. Thus:

$$\text{CN-7} \times 6c\text{-7} + \text{CN-8} \times 6c\text{-8} + 6c\text{-N} = \text{CN-6}$$
$$\text{CE-7} \times 6c\text{-7} + \text{CE-8} \times 6c\text{-8} + 6c\text{-E} = \text{CE-6}$$

and,

$$\text{CS-7} \times 6c\text{-7} + \text{CS-8} \times 6c\text{-8} + 6c\text{-S} = \text{CS-6}$$

In like manner, all evaluated corrections are substituted successively into the next preceding *c*-equation for the evaluation of the corrections at the next preceding junction. The same check is applied to all; namely, CN + CE − 1.0000 = CS. Note that in Equation (5*c*) there are no items in Columns (7) and (8); C7 and C8, therefore, do not enter into the computation of C5, as their multipliers are zero. Similar conditions are found throughout the remainder of the back solution, materially reducing the labor required for this part of the problem. When all the corrections have been computed and checked, back to C1, they are entered on the diagram on Fig. 1, as already explained.

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PAPERS

RELATION OF THE STATISTICAL THEORY OF TURBULENCE TO HYDRAULICS

By A. A. Kalinske,[1] Esq.

Synopsis

The basic principles of the statistical theory of fluid turbulence are outlined in this paper. Only those ideas and conceptions of importance in hydraulics problems are presented; the more complicated mathematical details are omitted. Turbulence can be characterized by intensity parameters which are the root mean square values of the fluctuating velocity components, and a length parameter proportional to the most probable eddy size. The specific parameters used in characterizing the energy dissipation and the diffusion power of turbulence are derived, and experimental techniques and data are presented to illustrate how these parameters are obtained. The two major problems in hydraulics in which knowledge of the turbulence mechanism is of importance are those involving the increase or decrease of energy dissipation, and the transportation of sedimentary materials. The practical application of the theories of turbulence to some specific problems is indicated. The experimental data were obtained by injecting color streams and suspended immiscible liquid droplets into turbulent water streams and by using motion-picture photography.

Purpose of Paper

Since about 1930, both in Europe and in the United States, considerable research on fluid flow has been concerned with the fundamental aspects of the mechanism of fluid turbulence. In aeronautics, wind-tunnel and atmospheric turbulence has been an important field of research since about 1925. Some of the findings of the aeronautics engineers and physicists have been permeating gradually into the hydraulics literature. Certain fundamental investigations on water turbulence have been conducted, and more and more are being started. However, all this talk and writing about turbulence generally leaves the practical hydraulic engineer quite uninterested. The practical usefulness,

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Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February 15, 1940**.

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either immediate or future, of knowledge relating to turbulence does not seem very apparent.

It is the purpose of this paper to correlate the knowledge of fluid turbulence with the solution of some practical hydraulics problems, and to present the information on fluid turbulence, already in existence, in a more simplified form. Some experimental data obtained by the writer and others will be presented, and various experimental techniques for obtaining pertinent information on liquid turbulence will be discussed.

WHAT IS TURBULENCE?

Turbulence, as it is present in wind-tunnel air streams, has been studied for some time and has gradually attracted the attention of hydraulics engineers, because there are many hydraulics problems which would be better understood if more were known of the mechanism of turbulence.

When observing turbulence by injecting a dye or some other material into water, or when observing smoke coming from a chimney, a most haphazard mixing process is seen, seemingly unpredictable in nature. Fundamentally, it may be said that a fluid at any point is in a state of turbulence if the direction and magnitude of its velocity vector vary with time. This variation is relatively rapid and in general cannot be predicted except in the probability sense. Turbulence may be caused as a result of high shearing stresses set up in a fluid in motion, by obstructions in the path of a fluid, by passing a fluid through a screen or honeycomb, or by some means such as mechanical stirring. Confusion has arisen in the minds of some regarding whether or not there is a definite periodic relationship between a velocity component at a single point with respect to time. True turbulence is of an entirely random nature without any kind of regular periodicity. A stream of discrete vortices which form directly behind a body immersed in a moving fluid, such as the von Kármán vortex street, is not true turbulence; it is referred to as vortex motion, which breaks down eventually into true turbulence.

Turbulence manifests itself in the form of large and small eddies, whirling about from one layer of fluid to another. These eddies can transfer momentum, mass, heat, etc., from layer to layer, and can also cause the high dissipation of energy that characterizes turbulent flow. It is apparent then that, to characterize various quantities regarding turbulence (such as, for instance, the size of the eddies, the velocity at a point, or the frequency of velocity fluctuation), average quantities must be dealt with and the principles of statistics employed in determining and in using these quantities. Additional fundamental ideas regarding turbulence can be found in the writings of H. L. Dryden,[2] Hunter Rouse,[3] Assoc. M. Am. Soc. C. E., and Boris A. Bakhmeteff,[4] M. Am. Soc. C. E.

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[4] "Mechanics of Turbulent Flow," by Boris A. Bakhmeteff, Princeton University Press, 1936.

BASIC PRINCIPLES OF STATISTICAL THEORY OF TURBULENCE

At the outset let it be noted that a complete and mathematically rigorous theory of turbulence has not as yet been developed. However, as far as the hydraulics engineer is concerned, such a theory, even if available, would be of little use. The mathematical difficulties are so great that the development of a rigorous analysis has taxed some of the best mathematical minds. The various ideas that have helped greatly in understanding the mechanism of turbulence are those developed by L. Prandtl,[5] G. I. Taylor,[6] and Theodor von Kármán,[7] M. Am. Soc. C. E. The basic statistical principles they have set forth have been checked experimentally, and are of fundamental importance in an understanding of the turbulence mechanism.

It is convenient to represent the varying velocity vector at any point in turbulent flow by means of the velocity components along three rectangular axes. The velocity in the direction of mean flow, which is taken along the x-axis, at any instant is represented as $(U \pm u)$, in which U is the mean velocity in that direction and u is the fluctuation in that direction. Obviously, the arithmetic mean of u is zero. The velocity along the other two axes, y and z, is represented as $(V \pm v)$ and $(W \pm w)$, respectively. If the mean flow is in the x-direction, V and W are zero. Since u, v, and w are quantities that vary with time, it is useful to represent them by certain statistical averages. Before this is done, the statistical distribution of these quantities should be investigated. That is, what is the form, for instance, of $f(v)$ which is a function such that $f(v_i)\,dv$ indicates the probability that the value of v lies between v_i and $v_i + dv$? Such a function, $f(v)$, is called a statistical distribution function. If the value of v could be determined at any point for a sufficiently long period of time the general form of $f(v)$ could be found.

The value of the velocity component v with respect to time is shown in Fig. 1. These data were obtained in a turbulent water stream at a point near the center of a 12-in. by 12-in. open channel where the mean velocity was 7.8 in. per sec (see heading "Experimental Technique"). A block diagram indicating the frequency of occurrence of various values of v is shown in Fig. 2. The smooth curve in Fig. 2 has the equation,

$$f(v) = y = \frac{1}{\sqrt{2\,\pi}\,\sqrt{\overline{v^2}}}\,e^{-a v^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which: $a = \dfrac{1}{2\,\overline{v^2}}$, to simplify typography, and $\sqrt{\overline{v^2}} = 0.725$ for the data in Fig. 1. The bar over a term or terms indicates a mean taken with respect to time.

This is the ordinary normal error law. Other investigators[8] have found the same type of distribution law, thus indicating the random nature of the velocity fluctuations. If the distribution function is of the same general form, different degrees of turbulence can be indicated by some average value of the quantities u, v, and w. The most significant average is that called the root mean square,

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[5] "Aerodynamic Theory," by W. F. Durand, Vol. III, J. Springer, Berlin, 1935, p. 162.

[6] "Statistical Theory of Turbulence, "by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 421.

[7] "Turbulence," by T. von Kármán, *Journal*, Royal Aeronautical Soc., Vol. 41, No. 324, December 1937, p. 1109.

[8] "Investigation and Analysis of Velocity Variation in Turbulent Flow," by L. F. G. Simmons and C. Salter, *Proceedings*, Royal Soc. of London, Vol. 145, 1934, p. 242.

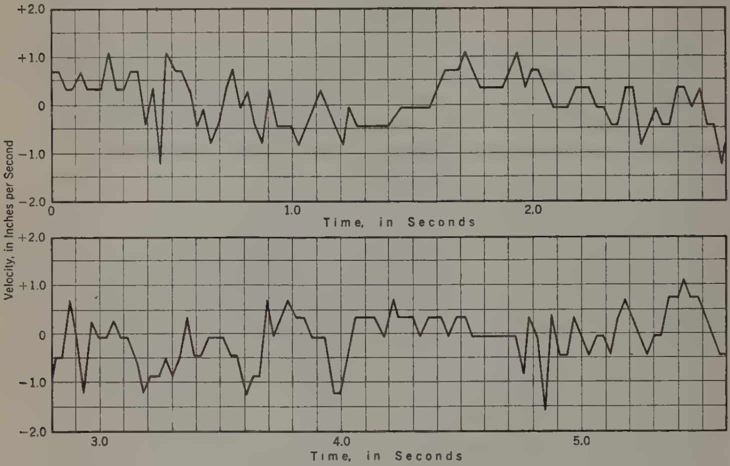

Fig. 1.—Transverse Velocity Fluctuation for Turbulent Flow in a Channel

or standard deviation, designated as $\sqrt{\overline{v^2}}$, for instance. The standard deviation indicates the spread of the distribution curve; also, its square, such as $\overline{v^2}$, is proportional to the energy of the turbulence for that velocity component.

Various types of turbulence can be designated depending on the relative magnitude and interrelationship of u, v, and w. The simplest type of turbulence, from the mathematical viewpoint, is that referred to as isotropic turbulence. In this type of turbulence $\overline{v^2} = \overline{u^2} = \overline{w^2}$, and also quantities of the type $\overline{u\,v}$, $\overline{u\,w}$, and $\overline{v\,w}$ are zero. These terms are referred to as mean products in statistics. If quantities such as $\overline{u\,v}$ are different from zero, a mean shearing stress and a mean velocity gradient exist in the fluid. In ordinary two-dimensional flow the mean shear stress, τ, is represented as,

$$\tau = \rho\,\overline{v\,u} \qquad (2)$$

in which ρ = unit density. A simple derivation of this is given by Messrs. Rouse[3] and Bakhmeteff.[4] Isotropic turbulence, therefore, cannot exist near a boundary. It is only approached in the center of large conduits and in artificially created turbulence, such as that beyond screens or grids. The turbulence in a hydraulic jump probably approaches isotropy.

Another fundamental concept that is useful in discussing turbulence is that of the correlation coefficient. In order to characterize a turbulent condition more completely, in addition to the quantities $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$, which characterize the intensity of the turbulence, a scale factor is necessary. Two different turbulent flows may have the same intensity, but the average size of the eddies may be entirely different. In order to identify the scale of the turbulence, Professor Taylor[6] has suggested that the instantaneous velocity components, either v or u, be measured simultaneously at two different points, along either the x-axis or the y-axis, over an appreciable period of time. Suppose v is measured simultaneously at two points a distance x apart. Let v_1 be the velocity at one point and v_2 that at the other point at the same instant, and define a number R_x such that,

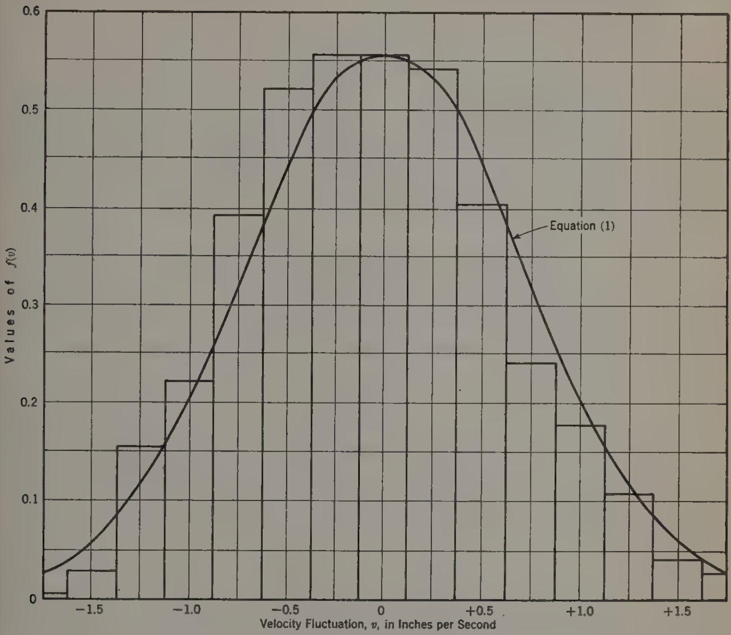

Fig. 2.—Frequency Distribution Curve for Transverse Velocity Fluctuation

$$R_x = 1 - \frac{\overline{(v_1 - v_2)^2}}{2\,\overline{v^2}} \quad \text{.......................} (3a)$$

Since $\overline{v_1^2} = \overline{v_2^2} = \overline{v^2}$ Equation (3a) can be transformed into

$$R_x = \frac{\overline{v_1\,v_2}}{\overline{v^2}} \quad \text{............................} (3b)$$

The term R_x is known in statistics as a correlation coefficient. Now, if the two points are close so that they are within the average size eddy, v_1 and v_2 will tend to be of approximately equal magnitude and of the same sign, and R_x

will be nearly unity. However, if the two points are an average eddy diameter or more apart, R_x will approach zero. Experiments verify this fact. Professor Taylor then defined a quantity, which is characteristic of the scale of the turbulence, thus:

$$l' = \int_0^x R_x \, dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which x = the distance when R_x approaches zero. Experimental values of R_x, as determined in the center of a 12-in. by 12-in. water channel where the mean velocity was 7.8 in. per sec, are shown in Fig. 3. Investigations of the variation of the correlation coefficient have proved helpful in the study of wind-tunnel turbulence, and should undoubtedly be useful in identifying various conditions of hydraulic turbulence.

Turbulence and Energy Considerations

First, the concept of energy of turbulence should be clearly understood. The kinetic energy per unit volume at a point in a turbulent fluid at any instant for the velocity component along the x-axis can be represented as $0.5 \frac{w}{g} (U \pm u)^2$. (The term w represents the weight of a unit volume of water.) The average kinetic energy is then $0.5 \frac{w}{g} (U^2 + \overline{u^2})$, since the arithmetic mean value of u is zero. The term $0.5 \rho U^2$ is the kinetic energy of the mean velocity. The energy represented by the other two velocity components is $0.5 \rho \overline{v^2}$ and $0.5 \rho \overline{w^2}$. Therefore, the total energy due to turbulence per unit volume is,

$$E_T = 0.5 \rho (\overline{u^2} + \overline{v^2} + \overline{w^2}) \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

If the turbulence is uniform in the direction of flow (that is, if it does not change, such as in a uniform pipe), the energy present due to the turbulence need not be taken into account in any energy equation. However, if the energy of turbulence varies from point to point, such as occurs in conduit expansions, beyond bends, through valves, etc., its omission, in any study of energy changes, results in an error. An important item regarding turbulent energy is that it is a form of degraded energy and will eventually be transformed into heat, and can never be recovered.

How does turbulence cause the dissipation of energy? All energy is dissipated into heat through the action of viscosity. If there were no viscosity, there would be practically no energy loss, and any turbulent energy created, such as in a sudden expansion, would tend to persist indefinitely. The high internal shear stresses set up by the turbulent eddies cause the large energy dissipation in turbulent flow.

In pipe flow, for instance, potential energy is being dissipated constantly into the form of heat. For turbulent flow, a simplified derivation for the rate of transference of the potential energy producing flow into energy of turbulence is as follows: Let the mean tangential shearing stress at a point in a circular pipe be τ. The mean difference in velocity between two layers of fluid is ΔU.

The work done per unit time by the force τ on an area $(\Delta x \, \Delta z)$ is then equal to $(\tau \, \Delta U \, \Delta x \, \Delta z)$. The work done per unit of time per unit volume of fluid is $\tau \frac{\Delta U}{\Delta y}$, or, in the limit, $\tau \frac{dU}{dy}$, in which $\frac{dU}{dy}$ is the mean velocity gradient. The mean unit shearing stress, τ, at a point distant y from the center of a pipe, of radius r is equal to:

$$\tau = \frac{h_f \, y \, w}{2 \, L} \qquad (6)$$

in which: h_f = loss of pressure head in pipe length, L, and w = unit fluid weight. This shear stress can also be represented as,

$$\tau = \mu \frac{dU}{dy} + \rho \, \epsilon \frac{dU}{dy} \qquad (7)$$

in which: μ = viscosity; ρ = unit density; and ϵ = mechanical viscosity due to turbulence (may vary from point to point). Except near the wall, in fully developed turbulent flow, the quantity $\mu \frac{dU}{dy}$ is very small in comparison to $\rho \, \epsilon \frac{dU}{dy}$. Thus in turbulent flow most of the potential energy is first transferred into turbulent energy at a rate represented by $\tau \frac{dU}{dy}$ per unit volume. The turbulent energy is then changed into heat by the action of viscosity. The rate at which potential energy is transferred into turbulent energy is not necessarily equal to the rate of energy dissipation into heat, at any one point. The turbulent energy can be diffused to other points by the action of the eddies, and then dissipated into heat.

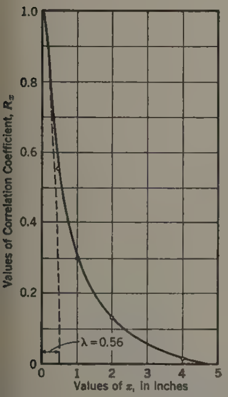

FIG. 3.—DISTANCE CORRELATION COEFFICIENT FOR TRANSVERSE VELOCITY, v

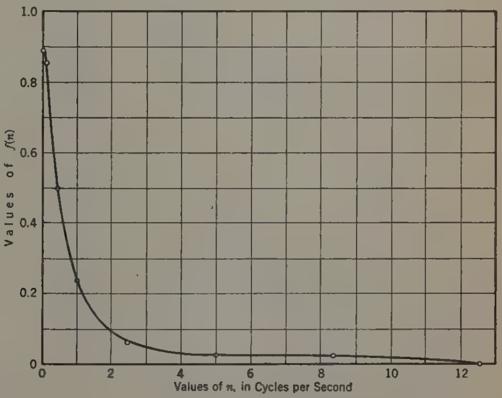

FIG. 4.—DISTRIBUTION OF TURBULENT ENERGY AMONG DIFFERENT FREQUENCIES

The mechanism by which turbulent energy is dissipated into heat is quite complicated, even in the simple case of isotropic turbulence. The rate of this energy dissipation is proportional to the viscosity times the mean square of the various instantaneous turbulent velocity gradients—that is, quantities such as $\mu\,\overline{\left(\frac{\partial v}{\partial x}\right)^2}$. The partial derivative sign is used because the component v also varies with y and z. The general formula for turbulent energy dissipation is given and discussed by Professor Taylor.[9] For isotropic turbulence Professor Taylor has shown that the mean rate of dissipation of energy per unit volume of fluid is:

$$K = 7.5\,\mu\,\overline{\left(\frac{\partial v}{\partial x}\right)^2} \tag{8}$$

Various simple relations exist between the different mean turbulent velocity gradients in isotropic turbulence. For instance $\overline{\left(\frac{\partial v}{\partial x}\right)^2} = \overline{\left(\frac{\partial u}{\partial y}\right)^2}$, and $\overline{\left(\frac{\partial v}{\partial y}\right)^2} = \overline{\left(\frac{\partial y}{\partial x}\right)^2}$.

One method of determining the value of a quantity such as $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ is by use of the correlation coefficient concept. Recalling the definition of R_x from Equations (3), if the values $\overline{(v_1 - v_2)^2}$ are obtained for small values of x, then approximately,

$$\frac{\overline{(v_1 - v_2)^2}}{x^2} = \overline{\left(\frac{\partial v}{\partial x}\right)^2} \tag{9}$$

Thus,

$$\overline{\left(\frac{\partial v}{\partial x}\right)^2} = 2\,\overline{v^2}\,\underset{x \to 0}{\text{Limit}}\left[\frac{(1 - R_x)}{x^2}\right] \tag{10}$$

Therefore,

$$K = \frac{15\,\mu\,\overline{v^2}}{\lambda^2} \tag{11}$$

in which $\lambda = \underset{x \to 0}{\text{Limit}}\left(\frac{x}{\sqrt{1 - R_x}}\right)$. The term λ is a length, and is a characteristic of the turbulence as far as energy dissipation is concerned. The value of λ for the data shown in Fig. 3 is 0.56 in. The value of $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ is then 3.32, since $\overline{v^2}$ was 0.52.

In analyzing the velocity fluctuation at a single point, the statistical distribution function was found to indicate the frequency of occurrence of various values of v, as shown in Figs. 1 and 2. It is of interest to make a similar type of statistical analysis of the distribution of the energy of the turbulence, such as $0.5\,\rho\,\overline{v^2}$, among the various frequencies. This will be useful in showing what is the average magnitude of the frequency of the velocity fluctuation. A function $f(n)$ is to be determined such that $f(n)\,dn$ indicates the most probable proportion of the total energy, as represented by $0.5\,\rho\,\overline{v^2}$, lying between the frequencies n and $(n + dn)$. More exactly, $f(n)\,dn$ does not indicate the fractional part of the total energy lying between the frequencies n and $(n + dn)$, but rather, the probability of the total energy lying in that range of frequencies. The method of the determination of $f(n)$, from experimental velocity fluctuation data, has been indicated by the writer in a previous paper.[10] It can be shown that,

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[9] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 430.

$$\overline{\left(\frac{\partial v}{\partial x}\right)^2} = \frac{4\,\pi^2\,\overline{v^2}\,\overline{n^2}}{U^2} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which $\overline{n^2} = \int_0^\infty n^2 f(n)\,dn$, since $\int_0^\infty f(n)\,dn = 1$. A plotting of $f(n)$ against n is shown in Fig. 4 for the velocity data shown partly in Fig. 1. The value of $\sqrt{\overline{n^2}} = 3.25$ and that of $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ is 3.56. This determination of $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ checks very well with the calculation by the correlation coefficient method as given by Equation (10). That the two methods are mathematically equivalent can be demonstrated. However, different data are used in each method to obtain the same final result.

Diffusion Characteristics of Turbulence

Turbulence eddies are capable of causing the exchange of momentum, heat, matter, etc., from one layer of fluid to another. Near solid boundaries the existence of a mean velocity gradient across a turbulent fluid indicates that momentum is being transferred from one layer of fluid to another. A temperature gradient indicates the transfer of heat; and a suspended material gradient in a turbulent stream indicates the transfer of matter. The characterization of a turbulent flow as to its diffusing or exchange power is an important item in any investigation of turbulence. The process of mixing in a turbulent fluid stream is often referred to as "eddy diffusion."

The laws of molecular diffusion have been applied to mass transfer processes in turbulent fluid streams.[11] However, the complete validity of this procedure is open to question, since the motion of fluid masses is not comparable to that of gas molecules. A common and convenient method of studying the diffusing power of turbulence is to observe the spread of heat, matter, or color from a point source of the heat, matter, or color injected in the turbulent fluid stream. Experiments were made by injecting a constant stream of color through a fine tube, and also by injecting colored droplets of a mixture of carbon tetrachloride and benzol, having the same specific gravity as water, and photographing the transverse spread of the color, or the droplets, with a motion-picture camera. The important quantity to determine from such experiments is the mean square cross-stream or transverse travel of the color at various distances downstream from the point of injection. Fig. 5 shows some sketches of typical motion-picture frames of the color and the droplets. The transverse travel at any distance x downstream is called Y and its mean square value $\overline{Y^2}$. Professor Taylor[12] has developed a theory relating x and $\overline{Y^2}$. His general premise is that the statistical correlation between the velocity of a fluid mass at one instant and at another later instant is high when the time interval is small, and this correlation approaches zero when the interval becomes large. This theory shows that $\overline{Y^2}$ varies as x^2 for small values of x, and gradually approaches a direct variation with x, as x increases. Professor Taylor[12] showed that

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[10] "Application of Statistical Theory of Turbulence to Hydraulic Problems," by A. A. Kalinske and E. R. Van Driest, Jun. Am. Soc. C. E., *Proceedings*, Fifth International Cong. of Applied Mechanics, 1938.

[11] "Studies in Eddy Diffusion," by W. L. Towle, T. K. Sherwood, and L. A. Seder, *loc. cit.*, 1938.

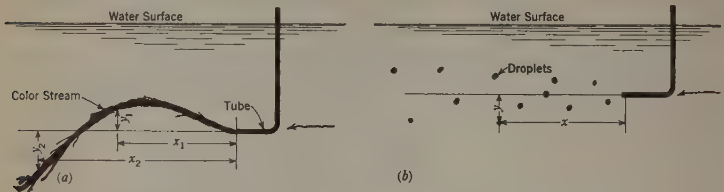

FIG. 5.—SKETCHES OF TYPICAL MOTION-PICTURE FRAMES, SHOWING COLOR STREAM, (a), AND COLORED IMMISCIBLE DROPLETS, (b)

$$0.5 \frac{d\overline{Y^2}}{dt} = \overline{v^2} \int_0^t R_t \, dt \text{ . . . . } (13)$$

in which: $\overline{v^2}$ = mean square transverse velocity fluctuation; R_t = correlation coefficient between velocity of a fluid mass at one instant and at an instant, t, later $= \frac{\overline{v \, v_t}}{\overline{v^2}}$; $t = \frac{x}{U}$; and U = the mean forward velocity. When R_t is near unity $\int_0^t R_t \, dt$ is equal to t and $\overline{Y^2}$ is given by:

$$\overline{Y^2} = \frac{\overline{v^2} \, x^2}{U^2} \text{ . . . . } (14)$$

When R_t approaches zero, $\int_0^t R_t \, dt$ becomes a constant, k, and $\overline{Y^2}$ is given by

$$\overline{Y^2} = \frac{2 \, \overline{v^2} \, k \, x}{U} \text{ . . . . } (15$$

Fig. 6 shows experimental data obtained by the writer on the relation o $\overline{Y^2}$ and x in a turbulent water stream near the center of a 12-in. by 12-in open channel at a point where the mean velocity was 7.8 in. per sec and $\sqrt{\overline{v}}$ was 0.78 in. per sec. Note the initial linear relationship of $\sqrt{\overline{Y^2}}$ and x; and at larger values of x, the linear relationship of $\overline{Y^2}$ and x. Note also that the initial slope of the $\sqrt{\overline{Y^2}}$-curve determines the value of $\sqrt{\overline{v^2}}$ according to Equation (14).

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[12] "Diffusion by Continuous Movements," by G. I. Taylor, *Proceedings*, London Math. Soc., Vol. 20 1921, p. 196.

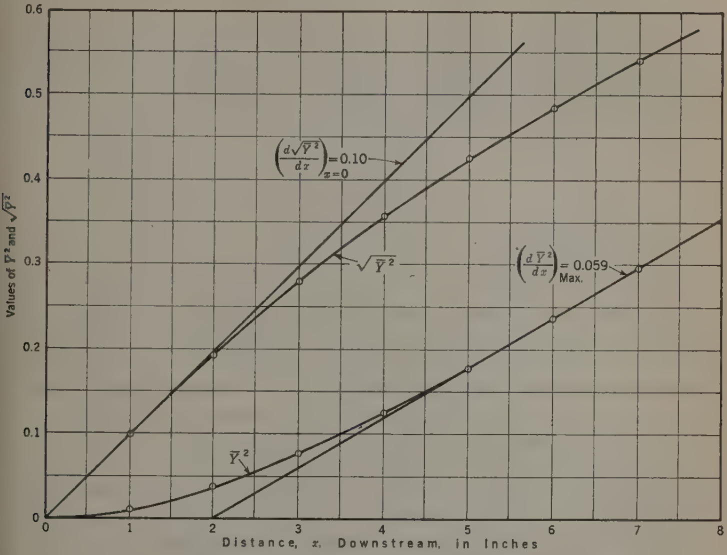

FIG. 6.—EXPERIMENTAL DATA ON DIFFUSION IN A TURBULENT WATER STREAM
(Bars Over Squared Terms Indicate the Mean of the Squares and not the Square of the Mean)

A mathematical relationship between $\overline{Y^2}$ and x is not a very convenient means of characterizing the diffusing power of turbulence. A diffusion coefficient would be the logical thing to determine. Using the ideas of molecular diffusion theory, such a coefficient would be defined as:

$$D = \frac{U}{2}\left(\frac{d\overline{Y^2}}{dx}\right)_{\max} \dots\dots\dots\dots\dots\dots\dots (16)$$

The value of $\left(\frac{d\overline{Y^2}}{dx}\right)_{\max}$ for the data in Fig. 6 is 0.059. The dimensions of D are $\frac{L^2}{T}$, or a velocity times a length. The velocity is the root mean square transverse velocity, $\sqrt{\overline{v^2}}$, and the length term is the length associated with mixing processes. This length term is proportional to the size of the turbulent eddies, and to Prandtl's so-called "mixing length."

It should be added that the relative concentration of the color or droplets transverse to the turbulent stream varied according to the normal error curve; that is,

$$N = \frac{1}{\sqrt{2\,\pi}\,\sigma}\,e^{-b\,Y^2} \dots\dots\dots\dots\dots\dots\dots (17)$$

in which: $b = \frac{1}{2\,\sigma^2}$, to simplify typography; and $\sigma^2 = \overline{Y^2}$, so that $b = \frac{1}{2\,\overline{Y^2}}$. The probability that a particle would lie between Y and $Y + dY$ is given by $N\,dY$.

A knowledge of the diffusing power of turbulence is most important in studies of suspended material transportation. The diffusion coefficient is proportional to the so-called coefficient of eddy viscosity, or mechanical viscosity, usually designated by ϵ, and occurring in the formula for shear stress caused by turbulent exchange of momentum:

$$\tau = \rho\,\epsilon\,\frac{dU}{dy} \quad \text{.......................} \quad (18)$$

The proportionality between ϵ and D must be determined experimentally.

EXPERIMENTAL TECHNIQUES

The different quantities which it is desirable to determine in order to characterize turbulence have been discussed. A more detailed explanation of the experimental methods that have been used and are available for the determination of these various quantities will be outlined briefly. The intensity of the turbulence is given by the magnitudes of $\sqrt{\overline{u^2}}$, $\sqrt{\overline{v^2}}$, and $\sqrt{\overline{w^2}}$ It is practically impossible to determine all three of these fluctuating components at the same instant. In aeronautics and wind-tunnel studies the hot-wire anemometer has become the standard instrument for measuring $\overline{u^2}$. The cross components, $\overline{v^2}$ and $\overline{w^2}$, can be determined with a special type of hot-wire anemometer.[13] The hot-wire anemometer has not been used very successfully in water, due primarily to difficulties such as collection of air bubbles and other foreign matter on the wire, the extreme magnification of the fluctuating current that is necessary in high-velocity water, and various other troubles Nevertheless, it has been used in water at lower velocities and undoubtedly will find further use.

One of the most convenient methods of determining simultaneous values of u and v, or u and w, in water, is by studying the motions of particles suspended in the flowing water. If illuminated, such particles make streaks on motion-picture film, and from the length and direction of the streaks two components of the velocity can be determined. Particles made from a mixture of carbon tetrachloride and benzine, having a specific gravity equal to that of water, have been used. The particles are illuminated by a narrow plane of light, so that only those particles in the light are visible.

Another method used by the writer to obtain values of the cross velocity v, for very short intervals of time, involves a thin color stream injected into the stream through a fine needle. A motion picture was taken of this stream close to the needle. The velocity v for very short intervals of time is obtained by measuring the transverse travel Y for short distances, x, downstream. Distance x is short enough so that the direction of the color stream is practically a straight line. The value of v is then $\frac{Y}{t}$, in which t is $\frac{x}{U}$. A large number of values of v enable the calculation of $\sqrt{\overline{v^2}}$. Such a color stream is shown in Fig. 5. The writer has found that good measurements of simultaneous values of u and v can be made by photographing, with a constant speed motion-picture camera, the travel of the immiscible droplets for a distance of about 1 in. from the point of injection. A magnifying type of close-focusing lens is used on the camera so that a picture is obtained of an area of about 2 sq in. Following the movement of the particles from frame to frame on the film permits the determination of u and v, and if sufficient data are obtained the quantities $\sqrt{\overline{v^2}}$, $\sqrt{\overline{u^2}}$, and $\overline{u\,V}$ can be calculated.

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[13] National Advisory Comm. for Aeronautics, *Technical Report No. 581.*

Another method of determining $\sqrt{\overline{v^2}}$ is from diffusion experiments, by finding the initial slope of the $\sqrt{\overline{Y^2}}$-curve, as shown in Fig. 6, and using Equation (14). The energy dissipative characteristics are determined by finding a factor proportional to the mean square of the turbulent velocity gradients such as λ, which was defined by Equation (11), or the factor $\overline{n^2}$, as defined in Equation (12). The term λ can be determined by measuring the correlation coefficient R_x, and the term $\overline{n^2}$ is calculated by analyzing the velocity fluctuation at a single point. In air flow R_x is determined by use of two hot-wire anemometers placed a distance x apart. The writer was able to measure R_x by injecting color at two points, a distance x apart, simultaneously, and photographing the two streams together.

The determination of the diffusion coefficient by diffusion experiments has been discussed. Undoubtedly, this is the most direct means of finding the diffusion characteristic of turbulence. Evidence exists that the diffusion coefficient is related to the intensity of the turbulence as given by $\sqrt{\overline{v^2}}$, and the length factor λ. The term λ is proportional to the size of the eddies, and the greater it is, the greater is the diffusion for a given value of $\sqrt{\overline{v^2}}$.

Unquestionably, the photographic method is an accurate and ideal method for making fundamental studies of turbulence in the laboratory. It is time-consuming and tedious, particularly the analyzing of the data recorded on the motion-picture film. An instrument that would give accurate recordings of the instantaneous velocity in a direction either longitudinal or transverse to the flow would be useful. Of course, such an instrument would have to be compared with data obtained by the photographic method in order to insure that it is following the velocity fluctuations accurately and is not introducing turbulent effects of its own. A forward step in the development of such an instrument has been made by C. W. Hubbard,[14] who has developed a so-called "angularity" indicator which measures the mean angle of flow. This angle, of course, is proportional to the mean transverse velocity.

The cup type of current meter is very sensitive to turbulence. However, because of its inertia it does not follow all the velocity fluctuations exactly. It does give some information regarding the magnitude of the velocity fluctuations and the size of the eddies, if the speed of rotation of the vane is obtained, by some automatic recording arrangement, for short intervals of time over an extended period.

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[14] "Investigation of Errors in Pitot Tubes," by C. W. Hubbard, *Preprint No. 3*, A. S. M. E., Annual Meeting, 1938.

Relation of Turbulence Research to Hydraulics Problems

Energy Dissipation.—There are many problems in aeronautics, physics, chemical engineering, and meteorology in which knowledge of the turbulence mechanism is quite important. However, this paper will be confined entirely to outlining some specific problems in hydraulic engineering which could be solved more accurately and more completely with information on turbulence.

Considering the general problem of energy dissipation, there are various individual problems which require a knowledge of the manner in which energy is dissipated. There are some design problems in which it is desired to obtain the maximum energy dissipation possible, and some in which the energy dissipation must be a minimum.

In a conduit of expanding cross section, the additional losses of energy associated with such an expansion are directly attributable to the creation of turbulence energy beyond that ordinarily present in uniform flow. In such an expansion, whether gradual or sudden, there is first an increase in the energy of turbulence and then a decrease.

In order to study the energy transformations completely, the energy of turbulence as given by Equation (5) must be found, in addition to the ordinary kinetic energy, as computed from the average velocity distribution. Fig. 7 shows some typical velocity data as obtained by the method of streak motion pictures in a 3-in. circular pipe. Fig. 8 shows similar data for a section of a 3-in. by 5-in. expansion. The rate of flow in both cases was 0.0818 cu ft per sec. In computing the turbulent energy the transverse velocity component, $\overline{w^2}$, was assumed equal to $\overline{v^2}$; this is quite true for flow in a circular conduit, except very near to the wall. The total mean kinetic energy as obtained from the mean velocity distribution is given by,

$$E_m = \pi \rho \int_0^r y \, U^3 \, dy \dots\dots\dots\dots\dots\dots (19)$$

The total energy of turbulence is

$$E_t = \pi \rho \int_0^r y \, U \left(\overline{u^2} + \overline{v^2} + \overline{w^2}\right) dy \dots\dots\dots\dots\dots (20)$$

The value of E_m in the 3-in. pipe is 0.255 ft-lb, and E_t is 0.00793 ft-lb. In this case E_t is 3.1% of E_m. The value of E_m in the expansion at the 3.75 diameter section is 0.157 ft-lb, and E_t is 0.0135 ft-lb, or 8.6% of E_m. Note the relatively higher value of the turbulent energy in the expanding portion. Data such as that given in Fig. 8 enable the accurate study of the energy changes taking place from point to point.

Although turbulent energy can be created very quickly, for instance, in a sudden expansion, the dissipation of that energy does not occur immediately. The expression "shock loss" is used many times to describe the energy loss at

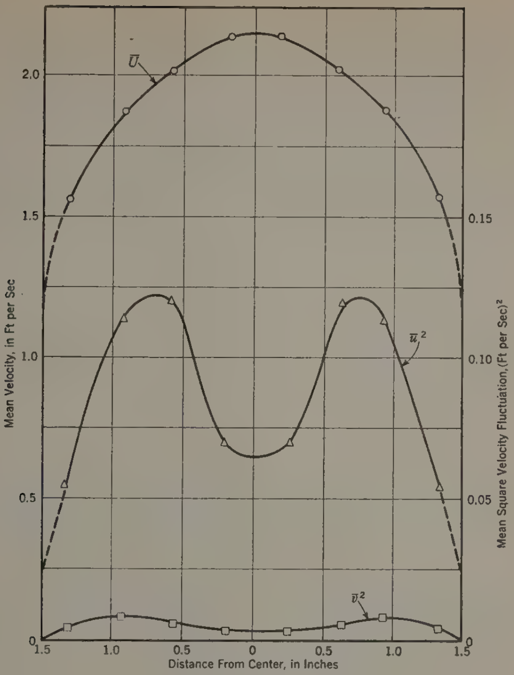

FIG. 7.—VELOCITY DATA FOR A 3-IN. UNIFORM CONDUIT

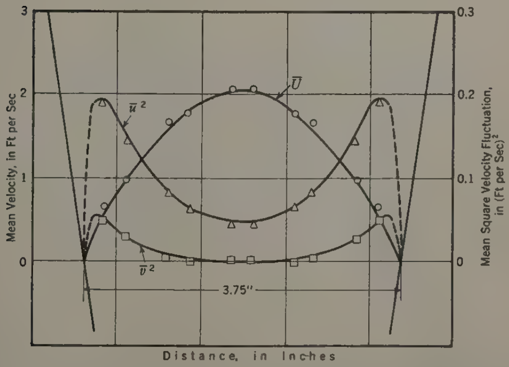

FIG. 8.—VELOCITY DATA FOR A SECTION IN A GRADUAL 3-IN. BY 5-IN. EXPANDING CONDUIT (Bars Over Squared Terms Indicate the Mean of the Squares and Not the Square of the Mean)

a sudden expansion. This is a misnomer and gives a wrong impression as to how the energy loss really occurs. The sudden change in velocity causes extreme turbulence; the energy represented by such turbulence is then gradually dissipated downstream by the action of viscosity. In the final analysis, viscosity, or friction, is the cause of the dissipation of energy into heat.

If it is desired to dissipate considerable energy, such as below dams or spillways, or to reduce the mean velocity of flow in steep chutes or conduits, much can be learned in regard to such problems from the theory of the turbulence mechanism. The rate at which potential energy per unit length of circular conduit is converted into turbulent energy is:

$$Q\,\frac{h_f}{L} = 2\,\pi \int_0^r \tau\,\frac{dU}{dy}\,y\,dy \qquad (21)$$

in which: $\frac{h_f}{L}$ = loss of head per foot of conduit; Q = total weight of water flowing in unit time; τ = unit shear stress at y distance from the center; and r = pipe radius. For fully developed turbulent flow the shear stress is equal to $\rho\,\epsilon\,\frac{dU}{dy}$, in which ϵ is proportional to the diffusion coefficient, or more specifically, to the quantity $\sqrt{\overline{v^2}}\,l$, in which $\sqrt{\overline{v^2}}$ is the intensity of the transverse turbulent velocity, and l is a length factor determined by the size of the eddies. Therefore, the rate of potential energy transformation can be increased by increasing ϵ, which can be done by introducing large-scale roughness so as to produce large, intense eddies.

For a given intensity of turbulence the smaller the eddies the higher the rate of energy dissipation will be, as is seen from Equation (11). The term λ will be smaller when the eddies are smaller, and therefore the average rate of energy dissipation will be greater. Thus, it appears that once the potential energy or mean kinetic energy of flow is in the form of turbulent energy, the dissipation of this turbulent energy can be hastened by the breaking down of the large eddies into smaller ones. This principle can readily be taken advantage of in the destruction of kinetic energy below dams and spillways. Thus, in converting potential energy into turbulent energy, such as for flow in a pipe or channel, the mean velocity can be reduced by the creation of intense large-scale eddies. To destroy the kinetic energy of high-velocity flow below a dam, it is desirable to produce intense, small-scale eddies.

TRANSPORTATION OF SUSPENDED MATERIAL

The relation of the distribution of suspended material to the exchange characteristics of turbulence has been quite well established.[15, 16] For equilibrium conditions the general statistical equation is:

$$\overline{v\,N} = c\,\overline{N} \qquad (22)$$

in which: v = instantaneous vertical velocity component; N = instantaneous

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[15] "Review of Theory of Turbulent Flow and Sediment Transportation," by M. P. O'Brien, M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1933, p. 487.

[16] "Mechanics of Suspended Material Suspension," by Hunter Rouse, *Proceedings*, Fifth International Cong. of Applied Mechanics, 1938.

sediment concentration per unit volume; c = average velocity of fall of single sediment particles in still water; and $\overline{N}$ = mean sediment concentration at any point above the stream bed. The left side of Equation (22) indicates the net transport of material upward by the turbulence, and the right side is the quantity that settles due to gravity. Under equilibrium conditions they are equal. The value of N at any instant can be represented as $(\overline{N} \pm \Delta N)$. Then the value of $v\,N$ at any instant is $(v\,\overline{N} \pm v\,\Delta N)$. It is quite apparent that when v is upward, ΔN will be positive, and when v is down ΔN will be negative, since the concentration of sediment in general increases from top to bottom of a stream. The mean value of $v\,N$ is then $+\,\overline{v\,\Delta N}$, since the mean value of $v\,\overline{N}$ is zero, because the arithmetic mean value of v is zero. The question now arises as to how large ΔN is. This depends on the distance from which the suspended material is brought up or down or, in other words, on the scale of the turbulence or size of the eddies. Introducing a length factor, l, characterizing the diffusing scale of the turbulence: $\Delta N = l\,\frac{dN}{dy}$; and Equation (22) becomes

$$\overline{v\,l}\,\frac{dN}{dy} = c\,\overline{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The term $v\,l$, which may be called β, is proportional to the diffusion coefficient defined in Equation (16). Whether β is identical to the momentum exchange coefficient, ϵ occurring in the shear formula, must be investigated experimentally. To date they have been assumed equal, and preliminary investigations indicate that they are identical. Integration of Equation (23) leads to a relation between the unknown sediment concentration, $\overline{N}$, at any point y above the stream bed, and a known concentration, $\overline{N}_a$, at a point a distance above the bottom of a stream or canal:

$$\text{Log}_e\,\frac{N}{N_a} = -\,c\int_a^y \frac{dy}{\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

This expression for silt distribution has been checked both in artificially created turbulence and also in natural streams. The term β varies vertically in a natural stream, tending to increase away from the bottom, reach a maximum at about the mid-depth, and then decrease as the surface is approached. However, the exact variation has not yet been determined.

Another important problem regarding turbulence and suspended material is the predication of the actual concentration at some point (for instance, just above the bottom), knowing the composition of the bottom, the various hydraulic factors, such as mean velocity and depth of flow, and the intensity and diffusing power of the turbulence. Such a relation is necessary if the expression for sediment distribution (Equation (24)) is to be of great practical usefulness. The mechanism of the picking up of sediment from the stream bottom by the turbulent eddies merits intensive experimental investigation. However, such an investigation will be of little fundamental value if the intensity and the diffusing power of the turbulence is not determined at the same time.

In the problems of silt sedimentation, the diffusion characteristics of turbulence are also extremely important. To illustrate how the theory of turbulent diffusion can be applied to the sedimentation of silt, two general problems will be analyzed.

Problem 1.—Consider the problem in which sediment is settling in a turbulent stream, and it is desired to know what the silt distribution is at various distances downstream from the point at which settling starts, and at which point the silt distribution is known. The assumption will be made that the turbulence is of such magnitude that most of the silt that reaches the bottom is not picked up again. Referring to Fig. 9, if the concentration at section (1) at a point h_1 ft above the bottom is N_1 units per unit volume for a particular size of silt, what will be the distribution of these N_1 units at some distance downstream? If the particles were weightless in water they would be diffused according to the laws of turbulent diffusion, and their distribution would be according to the normal error curve as given in Equation (17) (with $\sigma^2 = \overline{Y^2}$). In Fig. 9, the point $y = 0$ would be at the height h_1. Then the average concentration at point h_2 at the section downstream will be:

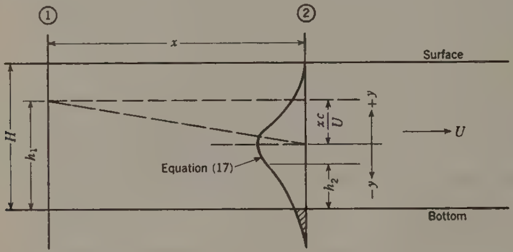

FIG. 9.—SEDIMENTATION IN A TURBULENT STREAM

$$\Delta N_2 = \frac{N_1}{\sqrt{2\,\pi}\,\sigma} e^{-bf}\,dy \quad \text{(25)}$$

in which: $b = \frac{1}{2\,\sigma^2}$; $f = (h_2 - h_1)^2$; and $\sigma = \sqrt{\overline{Y^2}}$ = the root mean square diffusion distance as measured from the initial elevation, h_1. If the concentrations N_1 and ΔN_2 are per unit volume, dy can be taken as unity. In words, then, ΔN_2 is the area under the distribution curve between $(h_2 + 0.5)$ and $(h_2 - 0.5)$ times N_1, the concentration per unit volume at h_1. The term $\overline{Y^2}$ is a definite function of x (see heading "Diffusion Characteristics of Turbulence").

Since the silt settles in still water at a velocity of c, it is apparent that the highest concentration (the point on the distribution curve where $y = 0$) will be found a distance of $\frac{x\,c}{U}$ below h_1, or, in other words, the distribution curve will be shifted downward a distance equal to $\frac{x\,c}{U}$. For simplification the velocity U used is the average velocity in the entire stream. The concentration at point h_2 will then be

$$\Delta N_2 = \frac{N_1}{\sqrt{2\,\pi}\,\sigma} e^{-bi} \quad \text{(26)}$$

in which: $i = \left[h_2 - \left(h_1 - \frac{x\,c}{U} \right) \right]^2$; and (as before) $b = \frac{1}{2\,\sigma^2}$, to simplify topography. To obtain the total average concentration per unit volume at h_2, values of ΔN_2 for all values of h_1, from 0 to H, must be summed:

$$N_2 = \int_0^H \frac{N_1}{\sqrt{2\,\pi}\,\sigma}\, e^{-b\,i}\, dh \qquad (27)$$

Note that N_1 is a function of h_1 and that h_2 and x are constants for each summation at any one point at section (2). Equation (27) gives the relation between the initial silt distribution and the distribution at another section downstream when settling is occurring.

Problem 2.—The other part of the sedimentation problem is that of finding how much silt settles in a distance x. Of course, the difference between the total concentrations at two points x distance apart is what has settled in the time required for the water to flow a distance x. However, this is a roundabout process of finding the quantity that has settled in the length x; therefore another method is used in finding this quantity. M. A. Velikanov[17] first presented the method outlined herein. The probable quantity of silt that will settle in a distance x from all the silt that passes a point h_1 in a given time is given by the shaded part of the area under the distribution curve, as shown in Fig. 9.

$$(\Delta M)_s = \frac{M_1}{\sqrt{2\,\pi}\,\sigma} \int_{-\infty}^{z} e^{-b\,y^2}\, dy \qquad (28)$$

in which: M_1 = total silt passing a unit area at point h_1 in a given period of time; and $z = \frac{x\,c}{U} - h_1$. The total silt that will settle in a unit time, in the reach x, is then given by

$$M_s = \frac{1}{\sqrt{2\,\pi}\,\sigma} \int_0^H M_1\, dh_1 \int_{-\infty}^{z} e^{-b\,y^2}\, dy \qquad (29)$$

Equation (29) can be simplified, especially if an approximate expression is sufficient. The probability integral can be evaluated readily by reference to books on probability and statistics. However, for the present it is deemed sufficient to present the basic equations only. The next important step is to conduct fundamental experiments in order to check these basic relations.

MISCELLANEOUS PROBLEMS

In addition to the general problems of energy dissipation and suspended material transportation, in which practical application is made of knowledge relating to turbulence, a few others may be stated, as follows:

(1) Many explanations have been proposed in the past as to the reason for the surface velocity in an open channel or stream being less than the maximum.

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[17] "Theory of Probability Applied to Analysis of Sedimentation of Silt in Turbulent Streams," by M. A. Velikanov, *Transactions*, Scientific Research Inst. of Hydrotechnics, Leningrad, 1936, p. 50.

The fundamental explanation is related to the turbulence mechanism, and more specifically to the process of diffusion of fluid masses in the form of eddies which are formed at the channel boundaries and then diffuse into the main body of the stream.

(2) The relation between high surface-water velocity and the entrainment of air is dependent on the degree of turbulence in the water, which in turn depends on the depth of flow and the channel roughness.

(3) A problem that has recently been studied in detail is that of the relation of the turbulence in a conduit to the accuracy of the Allen salt-velocity method of measuring discharge.[18] In general, the conclusions seem to be that for a high degree of turbulence (that is, for relatively high velocity) the method is more accurate than for slight turbulence. Errors may also result if the measurements are made in a reach of pipe in which the turbulence is changing, such as beyond a bend, valve, pipe expansion, or other turbulence producing device.

(4) Instruments for measuring mean velocity and static pressure in a turbulent water stream are affected more or less by the fluctuating velocities. Current meters, especially those of the cup type, greatly over-register in turbulent water. The velocity head read by a pitot tube is greater than that due to the average velocity depending on the degree of turbulence. The static pressure read by a static-pressure tube placed in a turbulent fluid has a reading greater than the true static pressure by an amount proportional to the mean squares of the cross-velocity components. However, with intensities of turbulence ordinarily present in water conduits or channels, the error in the reading of a well-designed pitot tube or static-pressure tube should be less than 1%.

(5) In conclusion, it should be mentioned that in aeronautics the measurement of the intensity, and the scale of turbulence, is rapidly becoming a standard laboratory procedure in all fundamental experimental work. In the near future, undoubtedly, such measurements will be made part of many hydraulic laboratory investigations. For example, if model studies are made of problems involving energy dissipation or of problems dealing with suspended material transportation or bed-load movement, in order that such studies be fundamentally sound, dynamic similarity must be maintained as far as intensity and scale of the turbulence are concerned. Turbulence should be thought of in terms of definite parameters instead of just as a qualitative descriptive term relating to a general condition of flow.

ACKNOWLEDGMENT

Grateful acknowledgment is given to E. R. Van Driest and to J. M. Robertson, Juniors, Am. Soc. C. E., research assistants, for help in taking and analyzing data.

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[18] "Contribution à l' Étude de la Mesure des Débits d'Eau par la Méthode Allen," by Martin A. Mason, Jun. Am. Soc. C. E., *Revue Générale de l'Hydraulique*, No. 24, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

EFFECTIVE MOMENT OF INERTIA OF A RIVETED PLATE GIRDER

By Scott B. Lilly,[1] M. Am. Soc. C. E., and Samuel T. Carpenter,[2] Assoc. M. Am. Soc. C. E.

Synopsis

The importance of the plate girder as a structural member has warranted many tests of its behavior under load. As an approach to the solution of this problem, a series of tests was planned to discover, if possible, how much the flange stresses in plate girders are affected by the rivet holes in the flanges. In other words, it is an attempt to find what, in this report, is called the effective moment of inertia, and, as far as the writers know, is the first attempt to secure such information. This report describes the methods used at Swarthmore College, at Swarthmore, Pa., for testing two plate girders and gives a resumé of the results. The intent was to obtain knowledge of the action of a plate girder under working conditions; therefore the girders were not loaded until they failed.

Both sets of tests show that the effective moment of inertia is very close to the gross moment of inertia, and that a good correlation of measured flange stresses with calculated stresses exists if the gross moment of inertia is used. Some data are also presented to justify the location of the neutral axis at the center of gravity of the gross cross-sectional area. Finally, the writers present a formula for effective moment of inertia.

Introduction

It is customary for structural designers to consider a net moment of inertia in the design of plate girders. For the purpose of this paper, "net moment of inertia" will be defined as the moment of inertia computed by deducting the effect of the material removed by holes in the tension flange. For clarity, "gross moment of inertia" is defined as the moment of inertia of the solid cross section of the plate girder, allowing no deductions for the effects of holes in the tension flange. The use of "net moment of inertia" in connection with the beam theory implies a displacement of the neutral axis that can be computed. The net moment of inertia assumes that holes on the gage lines of the tension flange are slotted for the full length of the girder. It is needless to say that the usual plate girder, having a rivet pitch in the flanges of from 2.5 in. to 6 in., does not represent a girder with slotted holes the full length of the tension flange. The practice of designers, who use the "gross moment of inertia" when computing the deflection of a plate girder, has been confirmed by some experimental data.

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To secure an answer to the question of whether the net moment of inertia or the gross moment of inertia, or an intermediate value which may be called the effective moment of inertia, should be used in the determination of the flange stresses of a plate girder, the same section of the same plate girder was tested under pure moment with the flange in the following condition:

Series A, solid flanges (no holes in outstanding legs of flange angles or in cover plates);

Series B, open holes in outstanding legs of flange angles and in cover plates;

Series C, with machine bolts in the holes described in series B; and,

Series D, with rivets in the holes described in series B.

It is evident that if the same section of the same plate girder is tested under the conditions described as series A, B, C, and D, any displacement of the neutral axis will be disclosed by the strains in the web and in the flanges.

TEST GIRDERS

The choice of test girders was predicated upon maintaining a depth-to-span ratio consistent with practice, at the same time selecting a size that could be handled readily in the laboratory. The test girder chosen has a depth-to-span ratio of 1 : 10.6. Fig. 1 is a detail drawing of the test plate girder, PT-1, and represents the girder in the state in which it was received at the laboratory. The holes marked "A" were drilled in the shop after tests A1, A2, and A3 had

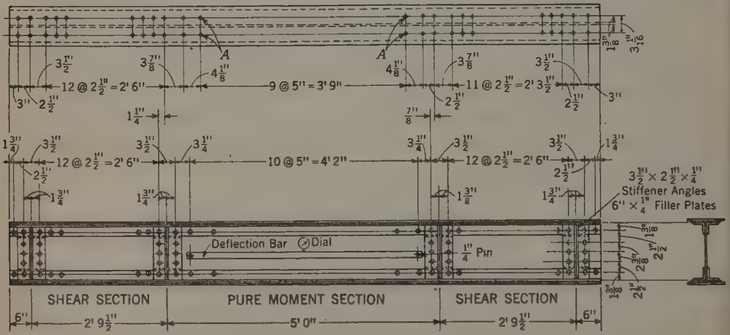

FIG. 1.—PLATE GIRDER PT-1; LOCATION OF DEFLECTION BARS AND SECTIONS OF PURE MOMENT AND SHEAR

been completed. Test girder, PT-2, was the same when received, except that the rivets connecting the web plate to flange angles in the pure moment section were spaced 2.5 in. apart. No holes were punched in the outstanding legs of the flange angles or in the cover plates in the center section of the girders. The makeup of the girders was the same throughout the full length, namely:

1. Web plate.........	12 in. by $\frac{5}{16}$ in.
4. Flange angles......	2.5 in. by 2.5 in. by $\frac{1}{4}$ in.
2. Flange plates......	6 in. by $\frac{1}{4}$ in.
2. Flange plates......	8 in. by $\frac{1}{4}$ in.

The depth, back to back of angles, was 12.5 in.; and, 8-in. cover plates were used so that strains could be measured on the side of these plates next to the 6-in. cover plate. This procedure gave no useful information and was abandoned.

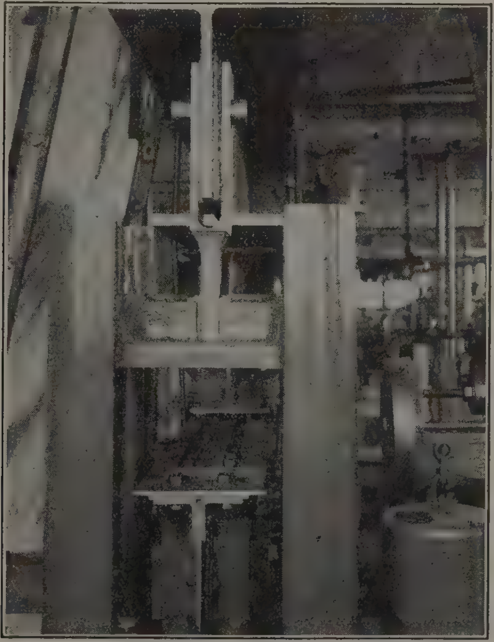

FIG. 2—VIEW OF TEST SET-UP, SHOWING THE FLANGE PLATE-CLAMPS IN PLACE

The "pure moment section" of the test beam was the only one in which the four comparative conditions of girder section could be secured. The cover plates were developed under the conditions of series A and B by bolts in the shear section only. Fig. 2 illustrates the typical installation of the flange clamps necessary to prevent the flange plates on the compression side from buckling during series A and B. The clamps were designed so that two $\frac{3}{8}$-in. set-screws came into point contact with the flange plates and two $\frac{3}{8}$-in. set-screws came into contact with the inner side of the outstanding legs of the flange angles along the gage lines. The plates were pulled into contact and held there by the clamps. Their construction was such that they added nothing to the flange area.

DESCRIPTION OF TEST SET-UP

The method of applying the loads was as follows: One end of the test girder rested upon a roller upon the platform of a 100,000-lb testing machine; the other upon a fixed cylindrical reaction point, supported by a steel trestle and adjusted to the same height as the platform of the testing machine. The load was applied to the test girder by an equalizer beam (see Fig. 3) through rollers situated at points 33.5 in. from the supports at each end of the test girder. (Test loads are noted in all cases, in this paper, as P and not 2 P. Therefore the bending moment in the pure moment section is always 33.5 P in-lb.) The load was brought to the middle of the equalizer beam by a loading beam. One end of this loading beam was connected to the movable head of the testing machine by a clevis and pin; the other end was attached to the corresponding end of the plate girder by two bolts, 1.75 in. in diameter, passing through steel slabs, top and bottom, contact being made through cylindrical reaction points. Contact between the loading beam and the equalizer beam was made by means of a roller. By this arrangement the concentrated load applied at each load point is exactly that which is weighed on the testing machine, making possible a total load of twice the rated capacity of the testing machine. In the presentation of data the testing machine load is the recorded value. In all cases, the total load on the girder is twice this value. The east end of the girder (see Fig. 3) rested on the testing machine.

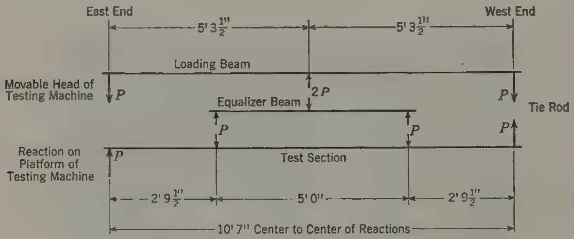
East End
West End
5' 3½"
5' 3½"
Loading Beam
Movable Head of Testing Machine
P
2P
P
Equalizer Beam
Tie Rod
P
P
P
Reaction on Platform of Testing Machine
P
Test Section
2' 9½"
5' 0"
2' 9½"
10' 7" Center to Center of Reactions

FIG. 3.—LOAD ANALYSIS FOR PLATE-GIRDER TEST

Pure Moment Section.—The method of measuring the deflections in the pure moment section was as follows: Two pins (0.25 in. in diameter), spaced $25\frac{1}{8}$ in. each side of the mid-point of the girder, were passed through the web plate in such a manner that the tops of the pins were level with the center line of the web plate. These pins extended through the web plate sufficiently far on each side so that they could support steel side bars $1\frac{1}{2}$ in. by $\frac{1}{4}$ in. in cross section. The holes in the ends of these bars were $\frac{3}{4}$ in. in diameter, and the distance between the supporting pins was 50.25 in. At the middle of the plate girder a third pin was placed through the web in a similar manner, and on each end of the pin was mounted an Ames dial, reading to $\frac{1}{10{,}000}$ in., with the contact points resting upon the side bars. (These dials are identified as the north and south dials.) Under this condition of loading, the middle of the test girder was subjected to a uniform moment, if the moment due to the weight of the girder itself is neglected. It is evident that the reading on the dials gave the deflection of the center line of the web below two points on that line which were spaced 50.25 in. apart (see Fig. 1).

To measure the stresses in the girder itself, gage points with 10-in. centers were drilled and readings taken with a 10-in. strain gage having a least reading of $\frac{1}{60{,}000}$ in. Assuming a modulus of elasticity of 30,000,000 lb per sq in., the least reading was equivalent to a stress of 50 lb per sq in. In the test procedure, readings were checked by independent observers until they agreed within six divisions on the dial, insuring readings with a maximum deviation of 300 lb per sq in.

Test Program

Fig. 4 shows the cross sections of the plate girder for each series of tests. In each case an initial load was applied and then the load was increased until the increment of deflection in the pure moment section was 0.0300 in., or slightly greater. After each series had been run, with loads to give the aforementioned deflection, the load was increased to 63,000 lb for series C-3 and D-3 for specimen PT-1, and readings were taken in the pure moment section. A detailed statement of the test program follows:

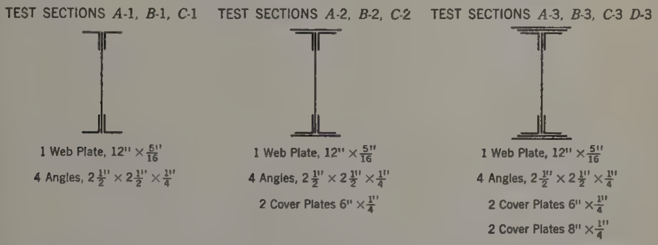
TEST SECTIONS A-1, B-1, C-1
TEST SECTIONS A-2, B-2, C-2
TEST SECTIONS A-3, B-3, C-3 D-3
1 Web Plate, 12" × 5/16"
4 Angles, 2 1/2" × 2 1/2" × 1/4"
1 Web Plate, 12" × 5/16"
4 Angles, 2 1/2" × 2 1/2" × 1/4"
2 Cover Plates 6" × 1/4"
1 Web Plate, 12" × 5/16"
4 Angles, 2 1/2" × 2 1/2" × 1/4"
2 Cover Plates 6" × 1/4"
2 Cover Plates 8" × 1/4"

Fig. 4.—Typical Cross Sections and Structural Make-Up of the Test Specimens

Test Section	Description (see Fig. 4)
A-1	Web plate with two pairs of flange angles riveted on, but with no holes in the outstanding leg.
A-2	6-in. cover plate added top and bottom. There were no holes in the plates in the pure moment section. The plate was held in contact with the flange angles by the clamps shown in Fig. 2.

Test Section	Description (see Fig. 4)
A-3	8-in. cover plate added to the 6-in. cover plate, top and bottom. In this case, also, the pure moment section had no holes in it and the plates were held in contact by the clamps.
B-1	Web plate and flange angles were tested as in test section A-1.
B-2	6-in. cover plates added, clamped, and tested as in test section A-2.
B-3	8-in. cover plate added to the assembly described in test section B-2, the two plates clamped and tested as in test section A-3.
C-1	$\frac{5}{8}$-in. machine bolts placed in the open holes ($\frac{11}{16}$ in. in diameter) of the outstanding legs, tightened with proper washers, and tested as in test specimen B-1.
C-2	6-in. cover plates added, bolted on, and tested as in test specimen B-2.
C-3	8-in. cover plate added to the assembly described under test specimen C-2 and tested as in test specimen B-3.
D-3	The two cover plates were riveted to the flange angles and tested as in test specimen C-3.

In series B (the section with open holes), the plate girder with the 6-in. and 8-in. plates attached was taken to a drill press and holes were drilled through the outstanding leg of the flange angles and the cover plates, in both compression and tension flanges. In specimen PT-1 the pitch was 5 in.; and in specimen PT-2 the pitch was 2.5 in. In series B, the holes were open in the pure moment section. (The pitch of the rivets in specimen PT-2 was first made 5 in. as in specimen PT-1, and tests were run which checked specimen PT-1 so closely that they are not reported herein. Then the girder was returned to the drill press and the additional holes drilled to make the pitch 2.5 in.)

Neutral Axis

Beginning with the assumptions of the common theory of flexure, it can be proved that the neutral axis must pass through the center of gravity of the cross section of a beam under flexure. This is substantially correct for homogeneous beams of the usual kind. However, in the design of riveted plate girders, the engineer must decide whether he will locate the neutral axis at the center of gravity of the gross cross section, or at the center of gravity of the net cross section, considering that the material that filled the holes has been removed from the tension flange, and that the holes in the compression flange are completely filled. If the net cross section existed at each section, as it would if it were obtained by slotted holes running the full length of the girder, and if the compression flange were solid, the center of gravity should undoubtedly be computed from the net cross section. The net section is not continuous, however, and it is questionable whether the rivets fill the holes of the compression flange sufficiently to consider the compression flange as solid.

That the neutral axis remains near the center line of the web plate is shown in Table 1, where data from tests on the two plate girders, PT-1 and PT-2, are presented. The observed results were obtained by measuring strains on 10-in. gage lines on both sides of the web plate. The gage lines were at distances of 1 in., 2 in., and 3 in. above and below the center line of the girder. A straight line was passed through the plotted data and the deviations obtained directly. The deviation of the neutral axis was also computed using the measured strains of the outside fibers of the girder, and these results are compared with those obtained from the web data. The experimental results all indicate that the neutral axis deviates only slightly from the center line of the web plate, and not as it has been considered to deviate by the designers using the net section. This means that the deformation of the compression flange is almost as great as that of the tension flange.

TABLE 1.—COMPARISON OF OBSERVED AND COMPUTED DISTANCES OF THE NEUTRAL AXIS ABOVE THE CENTER LINE, IN INCHES; PURE MOMENT SECTION

Description	(*a*) WEB PLATE AND FLANGE ANGLES			(*b*) WITH 6-IN. COVER PLATES . ADDED			(*c*) WITH 6-IN. AND 8-IN. COVER PLATES ADDED			
	A-1	B-1	C-1	A-2	B-2	C-2	A-3	B-3	C-3	D-3
Observed, Web Data:										
Specimen PT-1	0.19	0.15	0.14	0.16	0.08	0.07	0.09	0.04	0.05	0.10
Specimen PT-2	0.09	0.05	0.01	0.04	0.09	0.05	0.17	0.07	0.07	0.10
Observed, Flange Data:										
Specimen PT-1	0.11	0.12	0.07	0.11	0.05	0.11	0.09	0.05	0.03	0.14
Specimen PT-2	0.09	0.05	0.04	0.12	0.15	0.13	0.23	0.16	0.07	0.08
Computed*	0.34	0.63	0.63	0.25	0.68	0.68	0.18	0.66	0.66	0.66

* Computed on the basis of net section.

TEST COUPONS

Tensile tests were made by the National Bureau of Standards on eight test coupons from specimen PT-1, obtained from the extra pieces of material sent by the fabricator (see Table 2). The coupons had a reduced width of 0.5 in. and a gage length of 2 in. The coupons for specimen PT-2 were tested at Swarthmore College. The results were so similar that the data are omitted for present purposes. The average value of the modulus of elasticity was 30,000,000 lb per sq in.

OBSERVED EFFECTIVE MOMENTS OF INERTIA

In the pure moment section, the deflection in a length of 50.25 in. was read to the nearest 0.0001 in. by Ames dials. To observe values of the effective moments of inertia, the moments of inertia of the gross section, I_G (Column (2), Table 3), were assumed to be correct for series A, and a load, P_G, was applied until the deflection in the pure moment section was 0.0300 in. In series B, C, and D, loads were applied to each test section of the correct value to cause a deflection of 0.0300 in. Then it was assumed that since the deflection, Δ, of the plate girder was the same in each of these tests, and since $\Delta \propto \frac{P}{E\,I}$,

TABLE 2.—COUPON TESTS FOR TENSILE STRENGTH; SPECIMEN PT-1

Coupon No.	Coupon cut from:	Thickness, in inches	UNIT STRESSES, IN KIPS PER SQUARE INCH				Percentage elongation in 2 in.
			Young's modulus of elasticity	Proportional limit	Yield point	Tensile strength	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
B1	Web plate	0.299	30,100	40	42.0	57.8	40
B1-3	Web plate	0.300	30,000	37	42.0	67.1	36
C	6-in. cover plate	0.253	29,600	40	53.0	72.5	26
C-3	6-in. cover plate	0.253	28,200	40	54.1	73.8	26
D	8-in. cover plate	0.252	29,800	37	44.7	60.8	39
D-3	8-in. cover plate	0.250	31,200	37	43.0	60.8	38
V	Flange angles	0.240	29,300	37	43.6	58.8	38
V-3	Flange angles	0.241	27,900	39	50.4	62.6	31
Average	....	....	29,500	..	...	...	..

$$\frac{P_G}{P_o} = \frac{I_G}{I_o} \qquad \text{(1)}$$

and the observed effective moment of inertia in each case is

$$I_o = \frac{P_o}{P_G} I_G \qquad \text{(2)}$$

in which P_o and I_o are the load and moment of inertia for any particular section. For example, the load that caused a deflection of 0.0300 in. for section A-2 was 26,600 lb and the gross moment of inertia was 315 in.^{4}; the load that caused the same deflection for section B-2 was 25,200 lb. Therefore, the observed effective moment of inertia for section B-2 is $I_o = \frac{25{,}200}{26{,}600} \times 315 = 298.5$ in.^{4}

In this manner, values of the observed effective moment of inertia for each test section were computed and tabulated in Column (2), Table 3. In comparing the values in Table 3(*a*) and Table 3(*b*), it should be recalled that th pitch in this pure moment section was 5 in. for specimen PT-1 and 2.5 in. fo specimen PT-2. Instead of tabulating in Column (2), Table 3, the value o the gross moment of inertia opposite the test section of series A, the valu of I_o was computed from the value of the deflection, 0.0300 in., as determine by the moment-area method, using the value of E as determined from the tes coupons. These values check the values of the gross moment of inertia withi the limits of the experimental error and justify the foregoing procedure.

When the values of the observed effective moment of inertia are plotted the graphs show that they approach the values of the gross moment of inerti as the tests proceed from that giving the greatest deviation in series B (ope holes), through series C (bolted sections), to series D (riveted sections). The deviation is a very small value in series D—namely, 2.1% of the gross moment of inertia for specimen PT-1 and 7% for specimen PT-2.

TABLE 3.—EFFECTIVE MOMENTS OF INERTIA

Test section	MOMENTS OF INERTIA, IN INCHES^4				FLANGE STRESSES, IN KIPS PER SQUARE INCH									
	Observed	Computed			Observed		Computed, Using the Method Indicated							
		Gross section	Net section		t‡	b§	Observed moment of inertia		Gross section		Net Section			
			a*	b†							Neutral axis eccentric		Neutral axis at center	
							t‡	b§	t‡	b§	t‡	b§	t‡	b§
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(a) SPECIMEN PT-1														
A-1	189.0	192.9	178.5	179.5	16.9	17.6	17.3	17.9	17.2	17.2	17.6	19.6	18.5	18.5
B-1	182.0	192.9	164.3	167.3	18.3	19.1	17.9	18.6	17.2	17.2	18.2	22.3	19.5	19.5
C-1	187.0	192.9	164.3	167.3	17.7	18.2	17.8	18.2	17.2	17.2	18.2	22.3	19.5	19.5
A-2	315.0	315.0	300.9	301.6	18.0	18.7	18.1	18.7	18.4	18.4	18.5	20.0	19.2	19.2
B-2	298.5	315.0	270.7	275.5	19.3	19.7	19.2	19.5	18.4	18.4	19.2	23.7	20.8	20.8
C-2	306.5	315.0	270.7	275.5	18.9	19.6	18.6	19.2	18.4	18.4	19.2	23.7	20.8	20.8
A-3	494.0	490.8	476.9	477.4	18.5	19.0	18.7	19.3	19.1	19.1	19.3	20.3	19.7	19.7
B-3	460.5	490.8	430.0	436.0	19.9	20.2	20.2	20.5	19.1	19.1	20.1	24.4	21.5	21.5
C-3	472.0	490.8	430.0	436.0	19.7	19.9	19.8	20.0	19.1	19.1	20.1	24.4	21.5	21.5
D-3	480.5	490.8	430.0	436.0	19.3	20.2	19.1	20.0	19.1	19.1	20.1	24.4	21.5	21.5
(b) SPECIMEN PT-2														
A-1	189.0	192.9	178.5	179.5	17.7	18.1	17.0	17.5	17.2	17.2	17.6	19.6	18.5	18.5
B-1	177.0	192.9	164.3	167.3	19.4	19.8	18.7	19.0	17.2	17.2	18.2	22.3	19.5	19.5
C-1	179.0	192.9	164.3	167.3	19.0	19.2	18.5	18.6	17.2	17.2	18.2	22.3	19.5	19.5
A-2	315.0	315.0	300.9	301.6	17.9	18.6	18.1	18.7	18.4	18.4	18.5	20.0	19.2	19.2
B-2	282.0	315.0	270.7	275.5	20.0	20.9	20.2	21.0	18.4	18.4	19.2	23.7	20.8	20.8
C-2	295.0	315.0	270.7	275.5	19.4	20.3	19.3	20.0	18.4	18.4	19.2	23.7	20.8	20.8
A-3	500.0	494.0	480.1	480.6	18.5	19.8	18.1	19.4	19.1	19.1	19.3	20.3	19.6	19.6
B-3	442.0	494.0	433.2	439.0	20.6	21.0	20.8	21.8	19.1	19.1	20.1	24.4	21.4	21.4
C-3	453.0	494.0	433.2	439.0	20.0	20.9	20.5	21.0	19.1	19.1	20.1	24.4	21.4	21.4
D-3	460.0	494.0	433.2	439.0	20.3	20.3	20.2	20.6	19.1	19.1	20.1	24.4	21.4	21.4

* Neutral axis eccentric. † Neutral axis at center. ‡ Top flange. § Bottom flange.

FLANGE STRESS

In all of the test sections, flange stresses were obtained experimentally by measuring strains in the top and bottom flange. A 10-in. strain gage was used, with the gage lines placed symmetrically with regard to the longitudinal center line of the girder. Four gage lines were used for test sections A-1, B-1, and C-1, six gage lines for test sections A-2, B-2, and C-2, and eight gage lines for test sections A-3, B-3, C-3, and D-3. The layout of the gage lines on the 6-in. and 8-in. cover plates was such as to allow two gage lines to bisect the open holes in the flanges and later the bolts and rivets. The observed flange stress was computed from an average of the readings on all gage lines in every case, using the observed value of modulus of elasticity. The stresses were computed for a single load for each series, as follows:

Test sections	Load, *P*, in kips
A-1, B-1, and C-1	15.9
A-2, B-2, and C-2	26.6
A-3, B-3, C-3, and D-3	41.6

These particular values were selected because they caused an increment of deflection of 0.0300 in. in test sections A-1, A-2, and A-3 for specimen PT-1.

Using these loads the flange stress has been computed using the gross moment of inertia; using the net moment of inertia with the neutral axis at the center of the web plate; and finally using the net moment of inertia with the neutral axis at the center of gravity, considering the compression flange as solid and the tension flange as slotted.

The stresses computed by using the gross moment of inertia are compared with the observed stresses in Table 3(*a*), Columns (5), (6), (9), and (10). The computed flange stresses are somewhat lower than the observed stresses for all sections except for the initial solid flanges of series A. For those sections a good correlation exists. The stress in the compression flange is greatest for open holes, and gradually approaches the computed value as the method of holding the flange together is changed from clamps, to bolts, and to rivets. No such decrease is shown for tension flange. This probably indicates that bolts and rivets tend to relieve the stress in the compression flange.

In Table 3(*b*), Columns (5), (6), (9), and (10), the observed flange stresses are compared with the flange stresses computed using the net moment of inertia, with no shift allowed in the location of the neutral axis from the center line of the web plate. The computed flange stresses are about 2,000 lb per sq in. higher than the observed stresses throughout this comparison.

A comparison between the observed stresses and the flange stresses, computed by using the designer's net moment of inertia (see "Introduction") and assuming the neutral axis to be above the center line of the girder, in accordance with some design procedures, is shown by Table 3(*b*), Columns (5), (6), (7), and (8). The agreement between the compression flange stresses is very good. However, the computed stress for the tension flange is approximately 4,500 lb per sq in. higher than the observed. This would seem to indicate that this design procedure would give much larger flange areas than are needed to maintain safe stresses in the tension flange.

In addition to the foregoing information, Table 3 gives the flange stresses computed by the observed effective moment of inertia, using the observed deviation of the neutral axis as given in Table 1 from observed flange data. The excellent agreement between this column and the observed flange stress is an independent check on the observed results.

ACTION OF GIRDER UNDER HIGHER LOADS

After completing the tests where the loads were held to values giving a flange stress of about 20 kips per sq in., test sections C-3 and D-3 of specimen

PT-1 were subjected to loads causing flange stresses of 30 kips per sq in. The load was increased and decreased by increments, and strain readings were taken in the flange on both the up loading and the down loading.

The strain readings gave no indication of permanent set. A breaking of the scale around the rivet heads through the upstanding legs of the flange angles was noticed. A close observation showed a perfect set of orthogonal slip lines. No such breaking of the scale was noticed around the rivets through the flange plates, although this region was stressed higher. It is to be remembered that the holes in the cover plates were drilled, whereas the holes through the web and flange angles represented shop practice of punched holes. An investigation of one of these holes showed poor alinement of the plates and a very rough perimeter. Is it possible that the condition of the holes accounts for the observed strain lines?

STRESS CONCENTRATIONS

Stress concentrations, with their accompanying areas of plastic flow around the holes in the flanges of a plate girder, present some questions in regard to the validity and the interpretation of the strain gage readings explained herein. As treated in this paper strains have been reduced to terms of stress by applying Hooke's law. There may be objections to this procedure on two grounds: (1) That this does not represent the stress at any point; (2) that the average stress through the net section represents the true stress. In answer to these objections, the writers would point to Fig. 5, showing the way the stress varies across the net section. It is known that the maximum stress reaches the elastic limit, and further there is nothing that can be done about it; second, a unit stress arrived at by using a net area has no meaning.

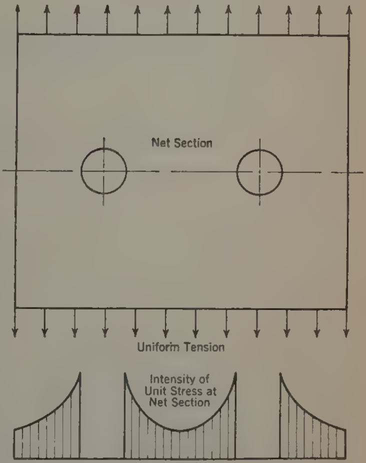

FIG. 5.—STRESS CONCENTRATIONS

It is reasonable to assume that any strain gage, operating on a 10-in. gage length over the holes, will register some plastic, as well as elastic, deformation. The writers do not know how to separate these components. A. Nadai states that the areas of plastic flow in the region of a hole are in the shape of a cross, the direction of the cross making an angle of approximately 45° with the principal stresses of tension or compression.[3] It is also well known that the intensity of stress is highest at the boundaries of the hole, which are on a diameter perpendicular to the applied principal stresses. Fig. 5 depicts this condition graphically. As the principal stress is increased, the stress at the boundary of the hole increases to the yield point value and plastic flow begins in the areas defined by the cross.

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[3] "Plasticity," by A Nadai, McGraw-Hill Book Co., Inc., New York ,N. Y., 1931, p. 103.

For any reasonable allowable design stress, the yield point stress will be reached at the hole, and plastic flow will occur. The designer's choice of 18 kips per sq in. instead of 20 kips per sq in. will not prevent this from happening, although it may limit, to some extent, the area over which plastic flow may extend.

Once a ductile material is strained beyond the yield point and the load is released, residual stresses are set up which will be beneficial in resisting the next applied load.

Effective Moment of Inertia

It seems reasonable to believe that both the strength and the stiffness of a plate girder are affected by the size and spacing of the rivet holes in the flanges of a plate girder. If the pitch were small enough there would be a slot the full length of the girder, and if the diameter of the rivet holes were large enough there would be no material to carry the load. The writers have developed the following formula which incorporates the results of the tests described in this paper, and which contains terms that include the effect of the diameter and pitch of the rivet holes.

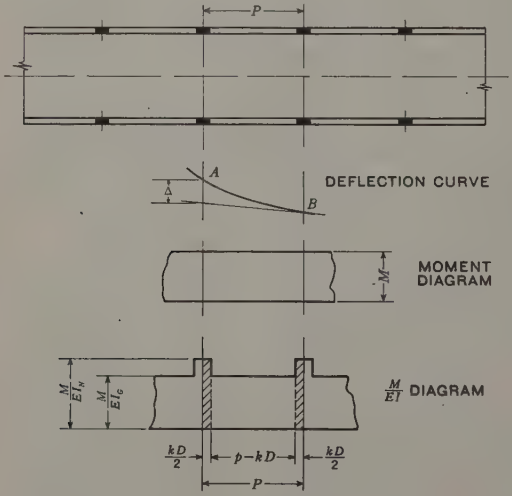

Fig. 6

Referring to Fig. 6, let L = length; M = moment; p = rivet pitch; D = diameter of rivet holes; $k\,D$ = the part of the rivet hole that is acting as a full net section; Δ = the deflection by point A with reference to a tangent to the elastic curve at point B; I_G = gross moment of inertia; I_C = clear moment of inertia (holes are deducted from the top and bottom flanges); and I_E = effective moment of inertia. The new term, I_C, the "clear" moment of inertia, has been introduced since these tests show that the neutral axis remains at the middle of the web plate. This fact is based on the practically equal stresses in the top and bottom flanges. By moment areas:

$$\Delta = \frac{0.5\,P\,M}{E}\left(\frac{p - k\,D}{I_G} + \frac{k\,D}{I_C}\right) = \frac{0.5\,P\,M}{E}\left[\frac{p + \left(\frac{I_G}{I_C} - 1\right)k\,D}{I_G}\right] \quad \text{. . (3)}$$

In general, $\Delta = \frac{M\,L^2}{E\,I}$ times a constant; and, in the present case:

$$\frac{L}{I} = \frac{P}{I_E} = \frac{P + \left(\frac{I_G}{I_C} - 1\right)k\,D}{I_G} \quad \text{. . . . . (4)}$$

That is,

$$I_E = \frac{I_G\,p}{p + \left(\frac{I_G}{I_C} - 1\right)k\,D} \quad \text{. (5)}$$

The quantity $k\,D$ in Equation (5) must be determined experimentally. The value of k probably depends upon D, the diameter of the hole, and g, the distance between rivet lines. The writers do not have enough experimental results available to determine this relation. It is thought that the value of k must lie between 0.6 and 1.0. The ratio of $\frac{I_E}{I_G}$ is called "m." Table 4 shows the value of "m" obtained from experimental data compared with the computed "m," using k equal to 1 and 0.6, respectively. It will be noted that the percentages of deviation of the computed value from the observed value change

TABLE 4.—COMPARISON OF VALUES OF $m = \frac{I_E}{I_G}$

Section	(a) SPECIMEN PT-1					(b) SPECIMEN PT-2				
	Value of m by experiment	$k = 1$		$k = 0.6$		Value of m by experiment	$k = 1$		$k = 0.6$	
		Value of m	Percentage deviation*	Value of m	Percentage deviation*		Value of m	Percentage deviation*	Value of m	Percentage deviation*
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
C-1	0.970	0.952	−1.85	0.972	+0.2	0.928	0.907	−2.3	0.943	+1.6
C-2	0.970	0.955	−1.5	0.972	+0.2	0.936	0.914	−2.3	0.954	+1.9
C-3	0.962	0.962	0	0.976	+1.5	0.916	0.927	+1.2	0.956	+4.4
C-4	....	....	....	....	....	0.930	0.927	−0.3	0.956	+2.8
D-3	0.980	0.962	−1.8	0.976	−0.4	....	....	....	....	....

* All deviations are computed from the experimental value.

from positive to negative with these values of k; that is evidence that k has a value between 0.6 and 1.0.

CONCLUSIONS

1.—The measured flange stresses are smaller than the computed stresses using a "net" moment of inertia based upon a deduction of rivet holes from the tension flange. In the case of the girder with a 5-in. pitch, the difference was considerable; with the girder having a 2.5-in. pitch there was a closer correlation. This was anticipated and furnishes some evidence that an effective moment of inertia should depend upon the rivet pitch as well as the rivet diameter. The net moment of inertia, used in design with top and bottom flanges alike, should yield conservative stresses.

2.—The experimental values of the effective moments of inertia are less than the computed gross moment of inertia. The effect of rivet pitch is shown clearly. However, the test girder having the 5-in. rivet pitch and riveted flanges gave nearly an exact agreement between the experimental and computed values of the moment of inertia.

3.—The tests indicate that the neutral axis should be assumed at the center of gravity of the gross cross section.

4.—The effective moment of inertia can be computed by using the theoretical expression, Equation (5), which involves the rivet pitch. Further experimental evidence is needed to establish this conclusion, and the value of $k\,D$.

5.—Since stress concentrations reaching the yield point must exist as soon as the unit stress in the gross section approaches the stress ordinarily specified for structural work, and nothing can be done to prevent them, the writers believe that design methods should be made as simple as possible. Any method that may be adopted must give sections that have been proved in practice to be satisfactory.

It is believed that the data from these tests point to the acceptability of the gross moment of inertia for design purposes. For ordinary girders, under usual loadings, it requires no modification; for cantilever girders, or girders with small pitch, it should be used with a coefficient such as is indicated by Equation (5).

It is hoped that this paper will stimulate discussion of this subject and reveal additional data which may confirm or disprove these findings. Many engineers may have an opportunity to measure strains and deflections of full-size girders. If this could be done widely, and such data carefully studied, all uncertainties would soon be removed.

ACKNOWLEDGMENTS

The investigators wish to acknowledge the cooperation of The American Institute of Steel Construction in conducting this research project. F. H. Frankland, M. Am. Soc. C. E., and all members of the Research Committee have contributed many suggestions that have been incorporated in this study. The writers are particularly indebted to A. R. Wilson, M. Am. Soc. C. E., engineer of bridges and buildings, of the Pennsylvania Railroad Company, for his sympathetic and timely discussions during the preparation of the paper.

Mr. D. F. Windenburg, physicist at the Bureau of Construction and Repair of the U. S. Navy Yard in Washington, D. C., has cooperated throughout the project. His assistance in making several tension tests on riveted plate specimens is acknowledged. Robert DeMoyer, Assoc. M. Am. Soc. C. E., a graduate student at Swarthmore and an associate professor of civil engineering at Pennsylvania Military College, at Chester, was of great assistance while performing the tests. Mr. Paul Kutz and Mr. G. A. Bourdelais, of the Swarthmore Engineering Division, are responsible for many of the details developed during the set-up of the testing equipment. The Belmont Iron Works, Philadelphia, Pa., performed the final riveting of the test girders free of charge.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ANALYSIS OF RUN-OFF CHARACTERISTICS

Discussion

BY OTTO H. MEYER, ASSOC. M. AM. SOC. C. E.

OTTO H. MEYER,[29] ASSOC. M. AM. SOC. C. E. (by letter).[29a]—The discussions have indicated a gratifying interest in this subject, and have directed attention to the need for further investigation, particularly on infiltration and seepage.

Mr. Cochrane attempts to derive the shape of the falling limb of the hydrograph by an inversion of the rising limb. The writer believes that the discharge, at any time after the time of concentration and after the cessation of the rain, must be determined by a function of outflow from storage. This is sufficiently well established by numerous studies of depletion curves[7, 8], and by the study referred to by Mr. Barnes. The sharp break at the top of the basic hydrograph, questioned by Mr. Cochrane, and also by Messrs. Zoch and Turner, is definitely verified by the hydrograph of Fig. 10 presented by Mr. Sherman. The writer has no criticism to offer on formulas for maximum flood discharge; however, these are outside the scope of this paper.

Mr. Barnes makes a timely comment as to the unreliability of the per cent run-off, or run-off factor. This factor is used where nothing better is available. As mentioned by Messrs. Sherman and Zoch, better results will follow from the deduction of infiltration from rainfall, and the consequent use of "net rainfall." However, infiltration and seepage are a separate subject, and consideration of them would complicate the subject unduly without affecting the characteristics being analyzed. In any case, a distribution of precipitation intensity giving a constant "net rainfall" is not impossible and would fit the assumptions on which the analysis was based.

The "subsurface storm flow" or "secondary base flow" mentioned by Mr. Barnes is an item of run-off of sufficient importance to merit considerable

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NOTE.—This paper by Otto H. Meyer, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. Victor H. Cochrane, and Bertram S. Barnes; March, 1939, by Messrs. LeRoy K. Sherman, Richmond T. Zoch, and Merrill Bernard; April, 1939, by Messrs. Franklin F. Snyder, and W. G. Hoyt; May, 1939, by C. O. Clark, Jun. Am. Soc. C. E.; and June, 1939, by Messrs. C. S. Jarvis, and Howard M. Turner.

[29] Asst. Engr., U. S. Engr. Dept., U. S. Engr. Office, Sacramento, Calif.

[29a] Received by the Secretary August 15, 1939.

[7] "Relation Between Rainfall and Run-Off from Small Urban Areas," *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equation (2), p. 151.

[8] *Monthly Weather Review*, September, 1934.

study. There can be no doubt that this type of run-off occurs and is often of considerable magnitude. The writer visualizes the source of this run-off as a sheet of water separated from the true "ground-water," possibly by a cushion of air filling the voids of the soil. Presumably, the same methods of analysis used for true surface run-off may be applied to a study of subsurface storm flow; however, it must be treated separately from surface run-off.

Mr. Sherman has apparently misinterpreted the hydrograph of Fig. 3, as no termination is indicated; the tail is intended to be of infinite length, only a portion being shown. As Mr. Sherman states, the use of unit hydrographs in combination, as shown by his example in Table 7 and Fig. 10, is a practical working tool, and the writer also uses this method, where applicable, on account of its simplicity. However, if the unit hydrograph used is that for a storm duration close to, or equal to, the time of concentration, inconsistencies may be expected to develop. For example, with a unit hydrograph for a one-day storm, if the tail is several days long, and the time of concentration is one day, a one-day rain produces a peak equal to the rainfall rate, but a two-day rain must necessarily cause the computed peak to be greater by the amount of the rate of discharge of the tail of the unit hydrograph one day after the peak of the unit hydrograph. This cannot occur in nature. The observed hydrograph of Fig. 10 obviously has the characteristics described in the writer's analysis, the time of concentration being about 25 to 30 min.

Mr. Sherman states that the time of concentration and the distribution for a three-day storm, as given in Appendix III, should be derived from the distribution for a one-day storm. This could be done, or, as in the example, the distribution for a one-day storm could be derived from that of the three-day storm, which in this case is the basic hydrograph, the time of concentration being three days. The process is reversible.

Mr. Zoch's distinction between run-off and rate of run-off, and between run-off and discharge, is commended. The writer does not agree with Mr. Zoch's statements on infiltration and internal drainage; however, that is considered outside the scope of this paper.

The writer cannot agree with Mr. Zoch as to the beginning of the depletion curve. It is true that with storms of short duration the part of the falling limb prior to the time of concentration contains both items of accretion and items of depletion; however, the succeeding portion must be pure depletion. This has been established empirically, and may be reasoned logically. The contribution to flow from every part of the basin has reached its maximum when the time of concentration is reached, and thus can only diminish. For the same reason, the maximum discharge can never be later than the time of concentration. The transition curve mentioned by Mr. Zoch must lie entirely between the end of the rain and the time of concentration. The "saturation curve" is not a part of the basic hydrograph, as it occurs only if the duration of the rain exceeds the time of concentration. The writer, therefore, adheres to the statement that the basic hydrograph consists of the two parts: (*a*) The concentration curve and (*b*) the storage, or depletion, curve.

Mr. Zoch states that the writer has not mentioned the determination of the time of maximum discharge. This time is indicated by the peaks in Fig. 2 and adequately described in the accompanying text.

Mr. Bernard calls attention to the lack of sufficient and suitable basic data for hydrologic studies. It is this very lack which forces hydrologists to use unsatisfactory expedients such as the run-off factor.

The writer would like to call attention to the fact that the definition of time of concentration appears in the second paragraph of the section on "Analysis of the Hydrograph." The mention of time of concentration in the "Synopsis" is not intended for a definition.

In describing, graphically, the sequence of occurrences on a water-shed during a storm, Mr. Bernard makes use of a method which could well be adopted generally in investigations of physical phenomena. Too often there are only the "rationalists," who go into transports of pure mathematics, and the "empiricists," who generalize from meager data. It is only by visualizing actual physical occurrences that engineers can be sure that their derivations are "on the right track."

Mr. Snyder's conclusion that the time of concentration is not the duration of a unit hydrograph is correct. The synthetic unit graph of Fig. 11 includes subsurface storm flow (secondary base flow), and is not all true surface run-off; the basic hydrograph of true surface run-off would reach a maximum of 6 245 cu ft per sec. A separate hydrograph of subsurface storm flow must be constructed if the writer's methods are used. The relative time to peak of the various hydrographs of Fig. 2, commented upon by Mr. Snyder, are subject to errors in drafting. The 1-hr rain would certainly cause a slightly earlier peak than a 7-hr rain.

The writer concurs thoroughly with Mr. Hoyt in his remarks on the necessity for research on the behavior of melting snow.

Mr. Clark ties flood routing and hydrographic analysis together in an admirable fashion. A complete hydrologic study of a large basin must always involve the construction of hydrographs of head-waters and tributaries and routing the combination. An integration of methods used in these two steps is of real value.

The basic hydrograph used to construct one hydrograph in Fig. 7 was synthesized from the unit hydrograph used for the other; the differences may best be understood from a careful study of the method of construction.

Mr. Clark's question on the apparent lack of effect of storage on the rising limb is best answered thus: The steepening of the slope of the water surface during rising stages opposes and nullifies the storage effect, whereas the flattening of the slope during falling stages increases the storage effect and thus makes storage the dominant factor.

In cautioning against indiscriminate use of the expression for time of travel, Mr. Clark has emphasized an important point. The variation of velocity along a stream has not been given the attention it deserves. No assumptions on this subject should be accepted without a most critical analysis.

The effect of land treatment as described by Mr. Jarvis must be considered in any hydrologic study. The writer has encountered this problem in attempt-ing to estimate run-off from irrigated lands, where the irrigation works cause almost complete retention.

Mr. Turner calls attention to the difference in shape of the writer's hydrographs for storms of the same volume but different intensity, and states that actual hydrographs are more similar. He shows, in Fig. 17, hydrographs resulting from storms of $\frac{6}{12}$, $\frac{8}{12}$, $\frac{10}{12}$, and the entire time of concentration. Actual storms lasting through the time of concentration are rare on large streams, and hydrographs resulting from storms of shorter duration than one-half the time of concentration are much more similar than those shown. The writer is of the opinion that the time of concentration of the Connecticut River at Sunderland, Mass., is about 5 to 6 days. Thus the hydrographs of Fig. 16 are not more similar than might be expected, especially as the shapes were probably determined largely by the precipitation occurring in the one or two maximum days of the storms.

No adverse criticism of the Committee on Floods of the Boston Society of Civil Engineers was intended, as the writer recognizes well the value of its work; it was to indicate the necessity for refinement that the shortcomings of the original formulas were mentioned.

In conclusion, the writer wishes to emphasize that it is the physical significance of the shape of the hydrograph, and not the empirical mathematical equations describing it, that is important.

Corrections for *Transactions:* Delete Footnote 17*b* (*Proceedings*, March, 1939, p. 514) and the proposed change to which it refers; in *Proceedings*, November, 1938, p. 1773, the sentence beginning in Line 29 should read: "It may also be used to synthesize the basic hydrograph from one produced by a storm of a known shorter duration"; in the caption to Fig. 2, change "basin" to "basic"; in Line 2 above Equation (17), delete "$\lambda =$ "; and change Equation (17) to read $\lambda = 0.5^{1/t_i}$. See also corrections published in *Proceedings*, February, 1939, p. 377; March, 1939, p. 526; and June, 1939, p. 1052.

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DISCUSSIONS

TESTS ON BUILT-UP COLUMNS OF STRUCTURAL ALUMINUM ALLOYS

Discussion

BY MARSHALL HOLT, ASSOC. M. AM. SOC. C. E.

MARSHALL HOLT,[14] ASSOC. M. AM. SOC. C. E. (by letter).[14a]—The writer wishes to thank Professor Niles for his discussion of this paper. As stated, a theory for columns that fail by twisting, and reliable equations for computing the strength of such columns, are not readily available. When satisfactory methods become available for predicting the strength of members that fail in this manner, the use of such members in special locations may become more common.

The prediction of twisting failure for four of the specimens in Table 3 was based on the following reasoning: The outstanding plates would fail by local buckling because the strength for this mode of failure is less than the strength if failure were to occur by lateral deflection of the specimen as a unit (compare the computed strength obtained by Equation (6) with the strengths obtained by Equations (4) and (5)). If the supported edge of the plate were free to rotate (but not deflect), the free edge of the plate would buckle into a single wave; but if the supported edge were fixed against rotation, the free edge would deflect into a number of waves, the number depending on the relative proportions of the plate. Since the torsional rigidity of these specimens is so small, the degree of fixation at the supported edge of the outstanding plate would not cause the development of more than one wave in the free edge. Thus, the center of the specimen would merely twist relative to the ends.

Failure under this type of action may or may not result in the collapse of the specimen but occurs at that load at which the lateral deflection of the free edge of the outstanding parts increases at a rapidly increasing rate (see Figs. 2(m) and 2(n)). The maximum load sustained by the specimen may be greatly in excess of this buckling load and, of course, will be accompanied by collapse of the specimen. There seems to be no method available for predicting the spread between the buckling load and the maximum load, but obviously it depends on the relative proportions of the cross-section. Considering failure in this light, the strengths of the specimens that failed by twisting were predicted within reasonable limits by Equation (6).

NOTE.—This paper by Marshall Holt, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Alfred S. Niles, Assoc. M. Am. Soc. C. E.

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[14a] Received by the Secretary September 13, 1939.

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DISCUSSIONS

TRAFFIC PROBLEMS IN METROPOLITAN AREAS

Discussion

By Earl J. Reeder, Esq.

Earl J. Reeder,[12] Esq. (by letter).[12a]—The discussions by Messrs. Herrold, Buckley, Mickle, Mitchell, and Coe amplified the original paper in a very effective manner. Although little difference of opinion has been expressed, some questions have been raised which will be treated herein.

Mr. Herrold is quite right in his statement that "the accident-prone driver presents a problem with which the careful driver cannot cope." There can be little doubt that the person who, through improper driving habits or unwillingness to conform with the requirements of traffic safety, becomes repeatedly involved in accidents is a menace to others who want to drive or walk safely. However, accident-proneness, as indicated by the repetition of accidents, does not appear to be a characteristic of a very large percentage of drivers. In fact, T. W. Forbes, of the Bureau for Street Traffic Research at Yale University, states[13] concerning a study of 29 500 drivers from the State of Connecticut for the years 1931–1936 that "We are thus left with the conclusion that not more than 1.3 per cent of the drivers were accident-prone and that only 3.7 per cent of the accidents in the states studied could be attributed to the accident-prone driver."

On the other hand, many street locations are definitely accident-prone as indicated by the repetition of accidents of almost the same kind. Analyses of many hundreds of collision diagrams of street intersections, and other problem locations, have shown definite patterns of accident experience at individual locations. In many cases these patterns are not at all similar to the traffic-flow pattern, for they show vehicles from certain directions predominating in the accident experience and those from other directions having little or no part in it. Surely, one cannot conclude that the careless drivers come from only one or two directions and that the drivers approaching from

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Note.—This paper by Earl J. Reeder, Esq., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. George H. Herrold, and Thomas Buckley, and April, 1939, by Messrs. D. Grant Mickle, Robert A. Mitchell, and Cleveland B. Coe.

[12] Chf. Traffic Engr., National Safety Council, Inc., Chicago, Ill.

[12a] Received by the Secretary September 15, 1939.

[13] *Journal of General Psychology*, April, 1939.

other directions are careful. Rather, the only conclusion that can be drawn in such cases is that there is something unusual and unanticipated about the condition to which the drivers and pedestrians are subjected from certain approaches. Further evidence of this is found in the fact that corrective measures often result in the prevention of these accidents by simplifying the traffic and making it easier to drive or walk safely.

Mr. Mitchell's interpretation of the statement "Contrary to the belief of many, a city street is not a continuous scene of human and mechanical disaster," is somewhat different from what was intended, and some explanation should be made. The purpose was not to convey the impression that a great many people over-estimate the seriousness of the traffic-accident problem. That certainly is not the case. It was intended, rather, to mean that many believe that accidents are rather indiscriminately distributed along the street and are likely to occur almost anywhere. In fact, however, they are concentrated at certain points and along certain sections of streets, whereas other sections are relatively free of accidents.

The difficulty with the belief of many, as the statement was intended to convey, is that it would place the responsibility for accidents too largely upon abnormal or unsocial drivers and pedestrians who are not interested in their own safety. However, the patterns of accidents at individual "high-accident" locations, and the freedom from accidents of long sections of street bearing the same traffic that becomes involved in accidents at other places, show quite conclusively that a substantial part of the responsibility for accidents lies with the conditions peculiar to individual locations.

The very basis of the engineering approach to the solution of traffic problems is the belief that the vast majority of drivers and pedestrians drive or walk safely to the extent that they can foresee and avoid hazards. The function of traffic engineering is to make it easier for drivers and pedestrians to use the streets and highways safely and expeditiously either by eliminating the hazards or by providing warnings, safeguards, and aids for avoiding them where they must remain. The more effectively this function is performed, the easier will be the functions of traffic-safety education and enforcement.

The field of traffic engineering has the very fascinating job of determining and evaluating driver and pedestrian practices as important factors in the design of traffic facilities, control measures, and safeguards. The more quantitative data that are gathered concerning traffic the more obvious it becomes that driver and pedestrian practices, under similar conditions, are approaching similarity. Vehicle speeds along a given section of street will fall into quite definite patterns, the dissenters from which are likely to be relatively few. Nearly all drivers will obey a "stop" sign where the computed "critical speed," as determined by view obstructions and other conditions, is very low. However, where the critical speed is relatively high, a much smaller percentage will obey because the others resent restrictions which are obviously unnecessary. These and other facts which traffic engineers observe enable this branch of the profession to design measures for the prevention of accidents and congestion and to obtain anticipated results.

Another important function of the traffic engineer is to make the results of his efforts to reduce accidents or congestion on existing streets or highways serve as the basis for foreseeing and preventing similar conditions in the design and construction of new ones. It is the assembly of that body of information concerning the value of different remedies for different problems and the conditions which differentiate one problem from another which will give to his proposals for avoiding accidents or congestion in new street or highway plans the prestige which comes from successful experience.

Fortunately, in setting up a new traffic-engineering department in a city, the engineer can start with treatment of the many accident or congestion problems that already confront him and work from them to the treatment of the problems which he foresees in the new streets or highways that are being planned. As time goes on, his job will change somewhat from the treatment of existing problems to the anticipation of those that will develop if the proper measures are not taken in the design and construction of improvements or new streets or highways.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

THEORY OF LIMIT DESIGN

Discussion

By E. S. Fabian, Assoc. M. Am. Soc. C. E.

E. S. Fabian,[52] Assoc. M. Am. Soc. C. E. (by letter).[52a]—The essential functional requirements of any structure are: (1) That it be strong enough to withstand the application of the loads of specified magnitude; and (2) that under the application of these loads it exhibit a certain degree of resistance to deformation and thus maintain its geometric integrity. Strength and stiffness within the prescribed limits are the basic considerations of any design theory whether applied to individual members or to entire structures. Since the indices of both the strength and the stiffness of any structural member are expressed in terms of its dimensions and the physical properties of the material of which it is made, strength and stiffness of a structural member are definitely related. Parametric equations for strength and for stiffness can be written in each case and, by varying the parameters, simultaneous values can be obtained from which curves can be drawn. From these curves the functional relation of strength and stiffness can be ascertained conveniently. It is the character of this function that determines whether the principle of superposition (see heading "Introduction") applies.

To admit the application of the principle of superposition to a structure, the following conditions must be postulated:

(1) The character of the impressed load must not change as the load is increased;

(2) The geometric configuration of the structure must not appreciably change with the increase in the load;

(3) As the Load P changes, the axial (and bending) loads on each member of the structure must change proportionally to the change in Load P;

(4) As the loads on the members change, with the change in Load P, the unit stresses and total deformations must remain proportional to the loads; and,

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Note.—This paper by J. A. Van den Broek, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. John H. Meursinge, I. K. Silverman, Edward Godfrey, Basil Sourochnikoff, E. Mirabelli, C. M. Goodrich, George Winter, and Francis E. Simpson; June, 1939, by Messrs. Joseph A. Wise, Alfred M. Freudenthal, Hans H. Bleich, Alfred S. Niles, and A. Floris; and September, 1939, by Messrs. L. H. Nishkian, and F. G. Eric Peterson.

[52] Associate Prof. of Mechanics, Univ. of Tennessee, Knoxville, Tenn.

[52a] Received by the Secretary August 2, 1939.

(5) The total deformations of the various members—tensile, flexural, and compressive—must be of the same order.

Conditions (1) and (2) must also apply to each member in the structure.

The author's predication (see heading "Introduction") "that a structure will fail only when as many members, or restraints, as the structure has redundants plus one, are loaded to their capacity strength" and his third dictum (heading "Theory") are unquestionably valid within the limits of the operation of the principle of superposition; so is his design procedure as illustrated by the application to the design of a tower. As the writer understands it, this design procedure consists of three main steps which can be outlined as follows: (*a*) Apply an over-stress factor, n, to the design load of a structure; (*b*) on the basis of this "capacity" load, draw a continuous force diagram, and if it closes, it may be regarded as satisfactory; and (*c*) assume the dimensions of one member, in conformance to the force diagram, and proportion the neighboring members in relation to it as indicated by the force diagram.

To design a structure in this manner on the basis of the configuration of the structure when it is unloaded, is to assume that this configuration will not change appreciably while the capacity load is gradually diminished to zero value and then gradually increased back to the $\frac{1}{n}$th portion of its capacity value. To assume that when this is accomplished each member (which was designed for its capacity load) will be subjected to $\frac{1}{n}$th portion of its capacity load, is to postulate that throughout the process of unloading and partial loading the loads on each member, its unit-stresses and its total deformations, are constantly proportional to Load P. It must be assumed that

$$\frac{dP'}{dP} = \frac{ds}{dP} = \frac{d\Delta}{dP} = \text{a constant} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which P = the total load on the structure; P' = the load on a member; s = the unit stress in the member; and Δ = the total deformation of the member. This assumption is equivalent to stating that the structure must act as a "unit"; that is, the structure acts as a whole and is analogous to a single member of like character. However, a condition of this nature can be approximated only in extremely rigid structures, in which the members at each joint can be assumed to be of virtually equivalent rigidities, giving the structure a character of homogeneity. Their axial deformations, whether tensile, compressive, or flexural, can be assumed to be very small, and the resultant displacements of the joints are negligible. The application of the principle of superposition can be admitted and the design can be made on the basis of static equilibrium. However, due attention to permissible joint displacements and consequent deformations has resulted, in the case of suspension bridges, in considerable economy of design.[53] Similar economy can undoubtedly be shown for other structures.

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[53] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 894.

The stiffness of a structure as a whole is a function of the ratio of the loads (dead and live) that the structure must carry to its over-all dimensions. In certain types of structures this ratio is rather small and, if the deflections are of no consequence and are permitted to be large, the design results in a "light" type of structure. The behavior of these structures under applied loads is very dissimilar to the behavior of structures of a heavy or rigid type. This latter type will display a degree of elastic stability under any condition of stress intensity in its members up to the yield point. (In certain cases elastic stability may persist when the stresses in some members are beyond the yield point.) The light type of structure, however, may reach a condition of neutral equilibrium, presenting the danger of failure by buckling, when the magnitude of stresses in its members is considerably below the elastic limit. This dissimilarity of action can be explained only by the supposition that the ratios of stresses to deformations are not the same in the two types. In a light structure the deformations proceed at a greater rate than the stresses, or, stated briefly, the principle of superposition does not apply. Such light structures are very sensitive to the effects of the eccentricities and nonsymmetries at the joints, and their light members will often be subjected to a combination of axial and lateral loads.

When applied to whole structures, the principle of superposition must be postulated from the proportionality of total deformations to the applied loads. In the manner indicated in the first paragraph of this discussion, it can be shown that, in general, the relation between the capacity loads of various structural members and their stiffnesses is not a straight-line function. It is an asymptotic curve, similar in many respects to Euler's curve, the asymptotic part of which is in the range of low slenderness ratios (that is, high stiffness) of the members. Assuming this part of the curve to be a straight line it can be seen that the axial deformations of heavy, stiff members, whether they are subjected to tensile, flexural, or compressive loads, will be of the same order. Experiments validate this assumption. As the curve recedes from its asymptote in the direction of higher slenderness ratios, the rapid change in the slope of tangents to the curve indicates that such similarity in axial deformations in slender members cannot be assumed. In other words, the principle of superposition cannot be applied to structures joined by slender members of heterogeneous character. For similar reasons the principle of superposition cannot be applied to slender members subjected to the simultaneous action of heterogeneous loads such as compressive and bending loads. It can be shown that under these conditions the load-resisting function of a member, as regards lateral loads, is approximately proportional to 1 minus the ratio of the applied axial load to the capacity axial load.[54] By analogy a similar proportionality can be postulated for the structure as a whole.

Where the conditions are such that the law of superposition cannot be applied, the distribution of loads to various members cannot be made on the basis of simple statical conditions—namely, the force diagram—but it must be made on the basis of Maxwell's equations—that is, by the "influence-line" method.

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[54] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 882.

Herein, to the writer's mind, lies the anomaly of the author's thesis. Although Professor Van den Broek emphasizes the importance of deformations, the force diagram is the key to his analysis; although he holds the deformations of a structure to be of prime importance, he criticizes, as unsound, the very methods of analysis which are based on considerations of deflections.

Just how far a theory postulated upon the law of superposition applies to structures to which the law does not apply, and within what limits of deviation from this law, is not a problem pertinent to this discussion of limit design. A theory pushed beyond its rational limitations becomes an hypothesis. That the standard specifications, formulas and methods used in structural design are open to questioning cannot be denied. They have been developed with reference to certain types of structures and to meet specific problems. Their application to the light type of structures, particularly of the highly indeterminate character, may be questioned on the grounds of economy. To this end, the author has rendered a valuable service in raising the problem. The theory of limit design is undoubtedly applicable in certain cases. With modifications, particularly with reference to compression members and the factor of safety, perhaps its application could be extended. The limits of its application and the extent of the necessary modifications can be ascertained, however, only by considerable additional analysis and by checks against the methods that experience has shown to be reliable and economical.

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DISCUSSIONS

HYDROLOGY OF THE GREAT LAKES
A SYMPOSIUM

Discussion

By Messrs. S. T. Harding, O. E. Meinzer, and R. W. Davenport

S. T. Harding,[18] M. Am. Soc. C. E. (by letter).[18a]—The two papers in this Symposium represent a hydrologic balance sheet for the Great Lakes. The method used is illustrated by its application to Lake Superior. The resulting values found for the individual items in such a balance have values which differ for some items from those more generally considered applicable to similar conditions. The authors' results can be reviewed both from the point of view of their detail procedure and from general comparisons of their results.

As Colonel Pettis states, a rigid solution is impossible where there are three unknowns and only two equations. He meets this condition by means of assumptions which provide the equivalent of a third equation. The correctness of his conclusions thus depends on the correctness of these assumptions.

Colonel Pettis assumes that the area supplying water to Lake Superior is limited to its topographic drainage basin. This assumption is the usual and natural one for this area. Its use requires that the sum of all items of water use must equal, and cannot exceed, the total rainfall on the topographic basin tributary to the lake.

The known items are combined in the expression $P + R - O - S$. Although all of these items cannot be measured completely or exactly they are determinable within reasonable limits. The greatest uncertainty is in the R item as Colonel Pettis states the run-off of only 49% of the tributary drainage has been measured.

The plotting of the average monthly values of $P + R - O - S$ (Fig. 1(c)) shows negative values for April and May. This means that the values used for the rainfall on the lake, the surface run-off into the lake and the lowering of the lake are not sufficient to supply the outflow from the lake in these months. This, in turn, means that some additional item of inflow is required to supply the outflow even if no evaporation occurs from the lake.

Note.—This Symposium was published in April, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1939, by A. A. Young, Assoc. M. Am. Soc. C. E.; and September, 1939, by Messrs. Alfred J. Cooper, Jr., and Adolph F. Meyer.

[18] Prof., Irrig., Univ. of California, Berkeley, Calif.

[18a] Received by the Secretary June 14, 1939.

Although the uncertainty in the values for P and R may account for the negative values found for $P + R - O - S$, all experience requires the inclusion of some material amount of evaporation in these months. This, in turn, results in requiring an additionally larger item of inflow from some unmeasured source.

Proceeding with the assumption that there is unmeasured ground-water flow into the lake, Colonel Pettis derives values for its amount which produce the results shown in his summary balance for both land and water areas. The curve for $P + R - O - S$, in Fig. 1(c) which is equal to $E - U$, or the total net value of the unknown evaporation from the lake and any other unknown inflow, indicates that either there are major omissions or errors in $P + R - O - S$ or that other sources of inflow must exist of appreciable amount. It also indicates that the values for evaporation by months must have a relative distribution through the year that is materially different from those that have been usually considered. If the unknown item of inflow is assumed to have a fairly constant distribution through the year, the resulting values for evaporation from the lake must have winter values that are larger than the summer values.

Using his assumptions Colonel Pettis completes the analysis to secure the resulting values for land losses, ground-water flow into the lake and evaporation from the lake. Just as Colonel Pettis used his results for $P + R - O - S$ to judge the amount of distribution by months of his unknown items, his conclusions can be used to judge the reasonableness of his results.

The rainfall on the drainage area must supply the land losses, the rainfall penetration which becomes the source of the ground-water and underground flow into the streams and the lake as well as the surface run-off. For Lake Superior alone the items of water supply must balance similarly. If one item is over-estimated some other item must be reduced correspondingly in the land balance. If inflow to the lake is over-estimated, evaporation from the lake will have to be increased.

Some of the limitations on the values that may be secured for evaporation from Lake Superior can be shown by varying the assumed value of the land losses for consumptive use. The total rainfall of 28.2 in. may be distributed in various amounts as illustrated in Table 6. These results illustrate the limitations on the different items. If 16.6 in. of the rainfall on the land are used in soil-moisture evaporation and plant transpiration, including rainfall interception by vegetation, and Colonel Pettis' estimate of stream flow is correct, there

TABLE 6.—DISTRIBUTION OF TOTAL RAINFALL

Description	Pettis	For Various Assumed Values		
Consumptive use, L, in inches of depth on land area	5.9	10.0	15.0	16.6
Inches of Depth on Land Area:				
Surface run-off, R	11.6	11.6	11.6	11.6
Underground flow into lake, U	10.7	6.6	1.6	0
Inches of Depth on Lake Area:				
Surface inflow to lake, R	17.9	17.9	17.9	17.9
Underground flow into lake, U	16.5	10.2	2.5	0
Resulting evaporation from lake, E	35.6	29.3	21.6	19.1

is no remaining supply available for ground-water flow into the lake. To meet Colonel Pettis' conclusions land losses can only be 5.9 in. and rainfall penetration would have to be 10.7 in. plus the ground-water portion of the stream flow.

Although no exact values can be set for these items, Colonel Pettis' conclusions differ materially from most of the present values used for these items. He secures a land loss of 5.9 in. where water surface evaporation is 35.6 in. or land losses are only one-sixth of water losses. The drainage area of Lake Superior contains many other lakes. Their total area is not available; if they represent one-sixth of the drainage area, their evaporation would require all of the water assigned by Colonel Pettis to land losses, leaving nothing for loss from the five-sixths of the area in land. For the vegetative conditions of this area, consumptive uses should exceed 5.9 in. The rainfall is sufficient to support active vegetation. Cropped areas would require more than 6-in. loss for usual yields; forest areas would consume in excess of this amount. Colonel Pettis finds a maximum monthly land loss of less than 1 in. per month. General values shown by available records are well summarized by Joseph Kittridge, Jr.,[19] who places the area surrounding Lake Superior in the group of annual water losses between 10 and 20 in. Mr. Kittridge found such losses to be less than 10 in. only in sagebrush and desert shrub areas. Although his method of deriving results includes unmeasured deep ground-water movement with the land losses, his values are derived from areas adjacent to Lake Superior where such deep ground-water movement is not a factor.

The reasonableness of the land-area balance can also be approached from another point of view. All the underground flow to the lake, plus at least part of the surface run-off, must represent rainfall that has penetrated below the plant roots and reached the water table. The portion of the surface run-off that represents ground-water flow into the streams is not discussed in numerical terms in the Symposium and would be difficult to estimate closely. As the stream flow is fairly well sustained throughout the year such ground-water flow appears to be a material part of the measured stream flow.

The quantity of the derived underground flow into the lake represents 37% of the precipitation. With ground-water movement into the surface streams, the summary presented by Colonel Pettis would require that more than one-half of the rainfall penetrate to the ground-water. This is a much larger proportion of rainfall penetration than has been found for other similar areas. The general results of observations of rainfall penetration in the Great Plains area[20] would indicate little penetration from the 28.2 in. of rainfall on heavy soils, to perhaps 6 in. on sandy lands if conditions are as favorable as on the drainage of the Loup River in Nebraska. The probable amount of rainfall penetration on the drainage area of Lake Superior would not appear to be sufficient to account for more than the ground-water flow into the streams which is included in the stream item of Colonel Pettis' water supply balance.

The small value found for consumptive use requires a correspondingly high value for the total run-off. For both surface and underground run-off, the total run-off derived is 22.3 in. from a rainfall of 28.2 in. This exceeds the quantity of run-off obtained from adjacent similar areas. Areas farther west in the same latitude, with about 20 in. rainfall, have a surface run-off of only 2 or 3 in., and have little rainfall penetration.

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[19] "The Magnitude and Regional Distribution of Water Losses Influenced by Vegetation," by Joseph Kittridge, Jr., *Journal of Forestry*, Vol. XXXVI, No. 8, August, 1938, pp. 775–778.

[20] "Direct Accretions to Ground-Water from Rainfall," by S. T. Harding, M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 18th Annual Meeting, 1937, pp. 368–371.

Evaporation.—The Symposium presents two determinations of evaporation from Lake Superior, separately derived. Their close agreement does not necessarily represent a proof of the correctness of either.

Mr. Hickman's method of observation of evaporation represents a commendable effort to eliminate one of the factors which causes observations with pans to differ from the loss from large water areas. The method used, however, raises additional questions regarding the interpretation of the pan results in terms of lake evaporation.

The maintenance of the temperature of the water in the pans at the same value as the temperature of the surface water in the lake does not of itself assure the same rate of evaporation. Although it may reduce the extent of the difference, comparisons between the temperature-controlled pans and measured evaporation from large areas are needed before the assumption that the controlled pan has a coefficient of unity in all months can be justified. Although the difference in temperature between the water in the pans and in lakes is one of the important factors in their different rates of evaporation, other factors such as rim effects of the pan, differences of exposure and differences in pan size still need to be considered.

Evaporation is dependent on the available heat supply. On the lake the heat supply is primarily that received from solar radiation. The evaporation will be less than the quantity of water that the total solar radiation can convert into vapor because of back radiation and any loss in heat to surrounding bodies. Heat is stored in the lake during months of rising water temperatures and withdrawn during those of lowering temperatures. The authors state that temperature variations extend to a depth of 180 ft. They present the monthly mean surface temperatures but do not show the monthly temperature profiles in the lake.

If the heat added to the pans during the cooling months, to maintain the same water temperature in the pan as in the surface water in the lake, is equal to the heat obtained at the lake surface by the cooling of the lake water, then the loss from the pan would represent the probable evaporation from the lake as far as the heat balance items are involved. In the summer months the heat removed from the water in the pan would need to be equal to the heat stored in the water in the lake. Without records of the heat added or removed from the pan, or of temperature profiles in the lake, comparisons on the basis of a heat balance cannot be made.

Mr. Hickman's results differ from direct observations on other lakes which do not freeze during the winter. On some western lakes conditions permit the same type of analysis used by Colonel Pettis where ground-water inflow or outflow is negligible in amount. Among such lakes in areas of fairly low winter temperatures are Pyramid and Walker lakes in Nevada. These lakes also do not freeze during the winter; both have sufficient depth to be comparable, in this respect, to Lake Superior. Results to 1935[21] show much less relative evaporation during the winter months than are found by the authors. The evaporation and air temperatures of the three lakes are compared in Table 7. For the air temperatures the U. S. Weather Bureau records at the nearest station are used, Lahontan being used for Pyramid Lake and Schurz for Walker Lake.

TABLE 7.—COMPARISON OF EVAPORATION AND AIR TEMPERATURES; PYRAMID AND WALKER LAKES IN NEVADA AND LAKE SUPERIOR

	Pyramid Lake		Walker Lake		Lake Superior		Percent of the Annual Total		
	Evaporation, in feet	Mean air temperature, in degrees Fahrenheit	Evaporation, in feet	Mean air temperature, in degrees Fahrenheit	Evaporation, in inches	Mean air temperature, in degrees Fahrenheit	Pyramid Lake	Walker Lake	Lake Superior
January	0.25	31.7	0.20	31.7	4.2	26	6.0	4.8	11.8
February	0.25	37.9	0.15	37.4	3.1	24	6.0	3.6	8.7
March	0.30	44.8	0.20	44.2	2.0	27	7.25	4.8	5.6
April	0.30	51.7	0.20	50.4	1.4	32	7.25	4.8	3.9
May	0.35	60.2	0.25	58.5	1.5	40	8.4	5.9	4.2
June	0.40	69.1	0.40	66.2	1.5	48	9.65	9.5	4.2
July	0.40	78.2	0.50	73.1	1.9	58	9.65	11.9	5.3
August	0.40	76.1	0.55	71.1	2.6	64	9.65	13.1	7.3
September	0.45	65.7	0.65	62.0	3.4	66	10.85	15.5	9.6
October	0.40	54.1	0.45	52.4	4.3	60	9.65	10.7	12.1
November	0.35	42.3	0.40	40.6	4.8	48	8.4	9.5	13.5
December	0.30	32.9	0.25	32.8	4.9	34	7.25	5.9	13.8
Total	4.15	53.7	4.20	51.7	35.6	...	100.0	100.0	100.0

Until the heat-controlled pan can be tested adjacent to a large water surface for which the evaporation can be determined, the coefficient that needs to be applied to the results with the heat-controlled pan must remain unknown. Although the coefficient might prove to be unity, both for annual and for monthly periods, the writer will venture the prediction that the loss from the heat-controlled pan will be found to exceed that from the adjacent large water area and that the monthly ratios of the pan to the large area will be variable. The size of the heat-controlled pan is also a factor in its amount of evaporation if its performance is similar to other types of pan. Comparisons with different sizes of heat-controlled pans would not be difficult to make; if size were found to be a factor it would demonstrate that the assumption of a value of unity for the coefficient of their pan as used by the authors was not justified.

Conclusions.—Colonel Pettis meets the apparent contradiction of negative evaporation in April and May, if the ground-water inflow to Lake Superior is taken as zero, by assumptions which remove that difficulty but lead to improbable values for land losses and rainfall penetration. Mr. Hickman's heat-controlled evaporation pans give values for evaporation which agree with Colonel Pettis' results but, if applied to Lake Superior without correction, lead to the same difficulties in land losses that result from Colonel Pettis' analysis.

Although these two papers represent a commendable effort to determine the hydrologic balance for the drainage area of Lake Superior, the problem does not appear to be solved. Further work needed appears to include: (1) A more direct study of land losses to determine what part of the rainfall may be available for ground-water movement into the lake; (2) perhaps a review and possible revision of the estimated surface flow from the unmeasured portions of the drainage area; (3) comparisons of the heat-controlled pans with large water areas whose evaporation can be measured; and (4) monthly temperature profiles in the lake to determine if cooling in the late fall and winter months can supply the amount of heat required for the estimated amount of evaporation in these months. Although previous estimates of evaporation from Lake Superior may have been too small, Mr. Hickman's estimate, when all factors are considered, appears to be too high. Actually the evaporation from the lake must have a value that can be supplied from the rainfall and leave a consistent water supply available for land losses. Mr. Hickman's results do not appear to meet this standard.

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[21] "Evaporation from Large Water-Surfaces Based on Records in California and Nevada," by S. T. Harding, *Transactions*, Am. Geophysical Union, 16th Annual Meeting, 1935, pp. 507–511.

O. E. MEINZER,[22] ESQ. (by letter).[22a]—The paper by Colonel Pettis does not discuss the geology and ground-water conditions of the drainage basin tributary to Lake Superior. The area is known to be underlain, for the most part, by pre-Cambrian crystalline rocks which in general are impervious or have only low permeability. In most places these hard old rocks are overlain by a thin mantle of glacial drift which is, in part, water-bearing.

So far as is known, the rocks of this basin do not, except locally, have any artesian structure or other structural features for conveying the ground-water laterally toward the lake. The ground-water is believed to be essentially under water-table conditions, with the water-table sloping toward the streams and branches and not generally toward the lake. If these conditions prevail, a large part of the stream flow is doubtless derived from ground-water, but the quantity of water percolating directly into the lake can be only a small fraction of the 10.7 in. postulated in the paper.

On the basis of the great mass of information that is available in regard to the geology and ground-water conditions of the United States, it can be stated confidently that, for most of the large drainage basins in the United States, the loss or gain of water by subterranean flow is relatively small, and the discharge by transpiration and evaporation approximates the difference between precipitation and stream discharge. In general, the rate of discharge by transpiration and evaporation changes gradually and systematically from one region to another with changes in climate.

Colonel Pettis gives the land losses for the drainage basin of Lake Superior as only 5.9 in. If a larger rate of loss is conceded for the extensive swamp areas and numerous minor lakes within the basin, the rate of loss from the actual land areas would be still smaller. If 10.7 in., the purported value of U, is added to 5.9 in., the purported value of L, the total is 16.6 in., which is approximately the discharge by transpiration and evaporation that would be expected in the Lake Superior basin from the known regional loss.

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[22] Geologist in charge Division of Ground Water, Water Resources Branch, U. S. Geological Survey, Washington, D. C.

[22a] Received by the Secretary August 18, 1939.

The writer has not been able to get a clear understanding of the process by which the evaporation from the lake (E) has been estimated or computed in this paper. It is apparently quite conjectural. The value of 10.7 in. for ground-water inflow (U) has been derived merely by subtracting $P + R$ from $O + S + E$. Furthermore, the value of 5.9 in. for the land loss (L) has been derived merely by subtracting $R + U$ from B. Apparently no effort was made to study the actual ground-water conditions.

It would seem that, before the results of 10.7 in. and 5.9 in. can be given favorable consideration, a systematic ground-water survey of the drainage basin should be made, including a quantitative ground-water study of at least one locality that appears relatively favorable for percolation of ground-water into the lake. The quantitative study should include such items as a contour map of the water-table and a determination of the extent, thickness, and coefficient of permeability of the water-bearing beds.

R. W. Davenport,[23] M. Am. Soc. C. E. (by letter).[23a]—From his analyses, Colonel Pettis reaches the conclusion that, for the drainage area of the Great Lakes, the evaporation and underground flow are greater, and the land losses are less, than the values that have been generally accepted.

The mean annual evaporation from Lake Superior has been estimated in notable investigations of recent years as follows: 22 in. by Mr. Freeman, 15 in. by Messrs. Horton and Grunsky, and 23 in. by Messrs. Hayford and Folse. These results contrast with 35.6 in. obtained by the authors of the Symposium. The substantial excess of the latter value of estimated evaporation over the others is impressive.

Unquestionably, the mean annual rainfall and the mean annual run-off from the land area, 28.2 and 11.6 in., respectively, as obtained by Colonel Pettis, may be accepted as reasonably reliable. The results for outflow from Lake Superior and changes in storage in the lake are determined by methods which are reasonably precise and should be correspondingly reliable. Consequently, Colonel Pettis' further conclusion that underground flow is more and losses are less than generally accepted seems to rest fundamentally on the validity of the values adopted by him for evaporation from the lake.

The writer has participated in rather extensive studies of so-called land (or water) losses, or the difference between rainfall and run-off, in Eastern United States. (The term "rainfall" as used herein includes all forms of precipitation.) The studies were supervised by Gordon R. Williams, Assoc. M. Am. Soc. C. E., under the general direction of the writer.[24]

They were limited to streams in the humid or semi-arid regions of the United States east of the Rocky Mountains. The basic period for which the computations were made is the water year, or year ending September 30. Only records which had continued without interruption for three years or more were used. The basins were also selected with a view to: (a) The avoidance of complications due to underground flow, as indicated by large springs or other evidence of material ground-water influence; (*b*) the avoidance of artificial regulation by storage; and (*c*) the availability of a sufficient number of precipitation stations for the reliable determination of the average precipitation in the respective basins. The report[24] gives results for about 200 areas with an aggregate period of record of more than 2 000 years, or an average of about ten years per record.

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[23] Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[23a] Received by the Secretary August 18, 1939.

[24] "Water Losses in the United States," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey (publication pending).

TABLE 8.—SUMMARY OF PRECIPITATION, RUN-OFF, AND WATER LOSS IN WESTERN ST. LAWRENCE RIVER BASIN AND ADJOINING REGION

Item No.	Basin above the indicated gaging station	Period studied, in water years	Drainage area, in square miles	MEAN ANNUAL QUANTITIES, IN INCHES		
				Precipitation	Run-off	Water loss
	(*a*) ST. LAWRENCE RIVER BASIN					
	Streams Tributary to Lake Michigan:					
1	Thornapple River near Caledonia, Mich..	1932–1934	773	32.0	9.3	22.7
2	Muskegon River at Newaygo, Mich.....	1932–1934	1 450	30.1	10.2	19.9
	Streams Tributary to Lake Huron:					
3	Tittabawassee River at Freeland, Mich...	1913–1914 1916–1920 1932–1934	2 530	29.7	9.3	20.4
	Streams Tributary to Lake Erie:					
4	River Rouge at Detroit, Mich..........	1932–1934	194	28.6	6.0	22.6
5	Huron River at Barton, Mich..........	1915–1920	723	31.7	9.2	22.5
	(*b*) MISSISSIPPI RIVER BASIN					
	Chippewa River Basin:					
6	Jump River at Sheldon, Wis...........	1916–1934	510	30.5	12.9	17.6
	Trempealeau River Basin:					
7	Trempealeau River at Dodge, Wis......	1915–1919	633	29.5	8.3	21.2
	Black River Basin:					
8	Black River at Neillsville, Wis..........	1915–1934	756	31.1	9.6	21.5
	La Crosse River Basin:					
9	La Crosse River near West Salem, Wis.	1915–1934	412	30.3	10.0	20.4
	Wisconsin River Basin:					
10	Rib River at Rib Falls, Wis............	1926–1934	309	30.1	12.4	17.7
11	Yellow River at Sprague, Wis..........	1927–1934	420	28.8	6.3	22.5
12	Kickapoo River at Gays Mills, Wis.....	1915–1933	......	31.6	9.3	22.3
	Rock River Basin:					
13	Sugar River near Brodhead, Wis.......	1915–1934	529	32.5	9.2	23.3
	(*c*) HUDSON RIVER BASIN					
14	Red River at Fargo, N. Dak............	1919–1923 1925–1933	6 420	20.8	0.6	20.3
15	Red River at Grand Forks, N. Dak.........	1882–1934	25 500	20.9	1.2	19.7
16	Red Lake River at Crookston, Minn.......	1922–1924 1926–1933	5 320	19.5	1.6	17.9

Table 8 shows a summary of precipitation, run-off, and water losses for areas in the western St. Lawrence River Basin and the adjoining region, as they appear in the report. The water-loss results are plotted in Fig. 10 in order to facilitate consideration in relation to their several geographical positions. The requirements of the study for continuity and duration of the records, freedom from complications caused by artificial regulation of stream flow, and adequacy of the rainfall data caused the distribution of selected basins to be sparse in the region indicated.

As used in these studies, the run-off is the surface stream flow measured at a gaging station, and the rainfall is the computed average precipitation over the drainage area tributary to the gaging station. The difference between these two factors, of course, represents the evaporation and transpiration from the basin plus ground-water outflow from the basin that is not measured at the gaging station, minus ground-water accretion from sources outside the basin which appears in the measured surface flow at the gaging station. The common designation of this difference as water losses, as representing the evaporation and transpiration, is a misnomer to the extent that underground flow is involved in the ways indicated. Adjustments to the water-loss results as computed would be subtractive for ground-water outflow that does not appear at the gaging station and additive for ground-water accretion from outside the basin that appears at the gaging station. Variations in evaporation, transpiration, and ground-water effects, both additive and subtractive, would be reflected in variations in the computed difference between rainfall and run-off.

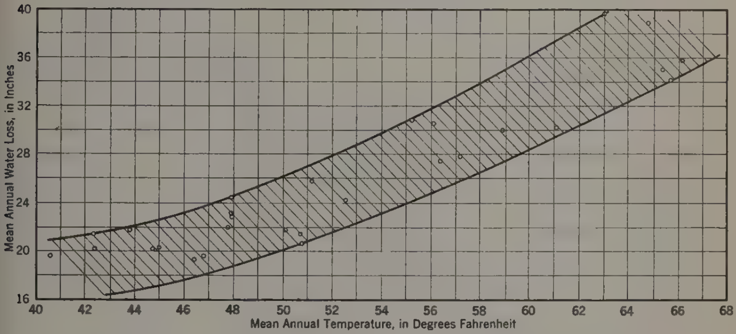

FIG. 10.—COMPARISON OF MEAN ANNUAL WATER LOSS AND MEAN ANNUAL TEMPERATURE FOR SELECTED BASINS WITH MEAN ANNUAL PRECIPITATION IN EXCESS OF 20 INCHES

The water losses show general uniformity under similar climatic conditions and a tendency to systematic transition in value as these conditions change. A striking feature is the progressive decrease in the computed loss in a direction from south to north. In the Gulf States computed losses of 30 in. to 35 in. or more are typical, whereas in the latitude of central New England and Wisconsin the losses seem to range generally from 20 in. to 24 in. In the aforementioned studies data were computed for comparing the mean annual water loss and the mean annual temperature for each of 28 drainage areas selected, with a view to adequacy of basic data and representative distribution.

Fig. 11 is a graphical representation of this relationship by means of 28 points plotted to show the respective basin determinations and lines enveloping them. It shows that for these 28 basins the water losses vary in general with the mean annual temperature. Unless one makes the unreasonable assumption that all of these basins are losing large parts of their water by underground flow, he must conclude that Fig. 11 shows approximately the losses by evaporation and transpiration in relation to the mean annual temperature.

The mean annual temperature of the area tributary to Lake Superior is of the order of 35 degrees. If the relation indicated by Fig. 11 is applicable, it would appear that the normal annual water loss by evaporation and transpiration is between 14 in. and 20 in. This compares with 16.6 in., as obtained by taking the difference between the rainfall and run-off results given by Colonel Pettis, and differs radically from the 5.9 in. computed by him.

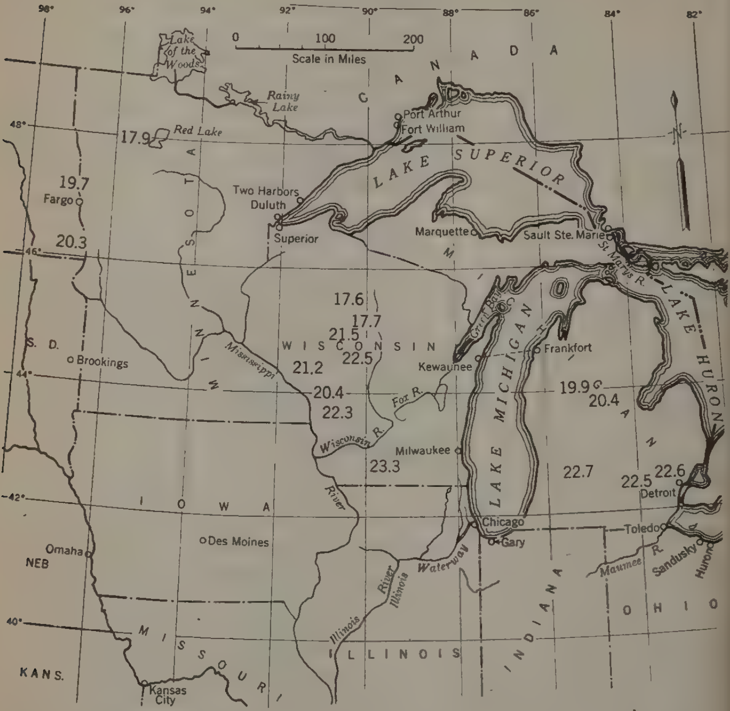

FIG. 11

Colonel Pettis' determination of evaporation seems to be a likely source of discrepancy, and it is hoped that the work on that factor may receive critical review.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

RECONSTRUCTION OF THE WALPOLE-BELLOWS FALLS ARCH BRIDGE

Discussion

BY LEWIS D. RIGHTS, M. AM. SOC. C. E.

LEWIS D. RIGHTS,[5] M. AM. SOC. C. E. (by letter).[5a]—The original bridge between Bellows Falls, Vt., and Walpole, N. H.—a 540-ft arch span across the Connecticut River, built in 1904—was an ambitious project years ahead of its time. When it was planned, the Advisory Engineer, the late J. P. Snow, M. Am. Soc. C. E., and the Consulting Engineer, J. R. Worcester, Hon. M. Am. Soc. C. E., both recognized the advantage of keeping the structure above possible ice damage; but the Bridge Committee, composed of local business men from both sides of the river, would not hear to the extra expense.

In 1906, a year after the bridge was finished, a flood damaged the bottom lateral bracing at one end and the consulting engineer recommended that a reinforced concrete slab be installed to brace the lower chord and protect the lower laterals, but there was no authority at hand to see that this was done.

In 1928, the dam was raised, making the pool 10 or 12 ft higher; the canal was enlarged and the power-house expanded. It was inevitable that sooner or later this elevation of the dam would cause a destructive flood.

At this time some strong State agency could have bargained with the power company to provide the necessary protection around the abutments as a partial offset for the benefit of increased power production. As is usually the case, the local authorities were incapable of handling these matters against a large power corporation, especially as the bridge was an interstate project. Had these precautions been taken, the heavy expense indicated in the paper might have been avoided. Fortunately, the entire bridge was not lost and the State of New Hampshire was able to take charge and repair it.

Offhand it would seem that the cheapest method of supporting the bridge during repairs would have been to use falsework towers as was done originally. So far as the writer knows the conditions were the same as when the bridge

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NOTE.—This paper by Messrs. H. E. Langley and Edward J. Ducey was published in April, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1939, by J. B. French, M. Am. Soc. C. E.

[5] Roselle, N. J.

[5a] Received by the Secretary August 14, 1939.

was first erected, except that the water was deeper in the pool. This would mean longer, and possibly a few more, piles but it would have been balanced by shorter timbers in the framing of the towers.

If the work had been done after the spring rise, when the ice was gone and before freezing weather, there would have been very little risk, even if a summer flood had occurred. Assuming that the erection cost of the bridge originally was one-half the cost of the superstructure, or $20 000, and taking into account the new repair material, it is difficult to understand the lump-sum bid of $120 000. On the "cost plus" plan the State assumed the risk from operating hazards, and if this had been done for the lump-sum plan this bid would probably have been much cheaper. Allowing for the changes in the abutments it would still leave a difference of $50 000 to $75 000 to be explained.

The writer hopes that the authors will give the costs of the repairs in detail; that is, the abutments, new repair steel, erection including cables and the new floor as well as information as to what was covered in the lump-sum bid.

All this does not in the least detract from the high praise due the engineers for the skill whereby the span was shortened from 540 ft to 486 ft and the spring-line pins raised 17 ft. The success of their scheme is the result of bold design, careful planning, and constant care.

The original bridge was erected in upper New England, over a capricious river, in the depth of winter because of the delays of the Committee in awarding the contract. The bridge was repaired, also, in the winter months, probably due to the delays of the "powers above" in reaching decisions. It is to the glory of engineers that they are willing to accept difficult and hazardous conditions forced by circumstances beyond their control, without wasting time quibbling about them, and that they then proceed to overcome these hazards successfully.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

DESIGN OF AN OPEN-CHANNEL CONTROL SECTION

Discussion

BY FRANK S. BAILEY, ASSOC. M. AM. SOC. C. E.

FRANK S. BAILEY,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—The author seems to have assigned himself a difficult task and performed it very ingeniously. The writer's first impression of this paper was that it was too complicated for his understanding and possibly in this case the first impression was the best. However, during a leisure hour he began trying to follow the author's reasoning and to check the equations he has derived.

Becoming puzzled over Equation (6) he wrote a letter of inquiry to the author, who replied courteously:

> "* * * if (5) is equated to 0, we find that this equation is satisfied if any of the factors are equated to 0. In other words, $Q - A_c^2\, g\, m = 0$. I did not think it necessary to call attention to the other mathematical possibilities which have no practical significance to our problem, for example when $A_c = 0$."

Although it is probably true, as Mr. Kennison states, that $Q - A_c^2\, g\, m = 0$, the writer has not found why it must be so. He believes that

$$Q^2 - g\, A_c^3\, f'(A_c) = 0 \qquad (46a)$$

and that

$$Q^2 = \frac{g\, A_c^3}{B_c} \qquad (46b)$$

Equation (5), obtained by differentiating Equation (4), could read

$$\frac{dQ}{dA_c} = \frac{Q^2 - g\, A_c^3\, f'(A_c)}{A_c\,(Q - A_c^2\, g\, m)} \qquad (47)$$

in which $f'(A_c)$ signifies the derivative of $f(A_c)$ with respect to A_c. Perhaps Equation (5) is just as valid; but Equation (47) seems more complete. Re-

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NOTE.—This paper by Karl R. Kennison, M. Am. Soc. C. E., was published in May, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1939, by Messrs. V. L. Streeter, Emery H. Willes, and Robert O. Thomas.

[9] East Braintree, Mass.

[9a] Received by the Secretary July 29, 1939.

ferring[10] to Fig. 4, it is evident that $\Delta A_c = B_c \, \Delta d_c$; and, in the limit, $\frac{dd_c}{dA_c} = \frac{1}{B_c}$, which is the derivative of $f(A_c)$. Therefore, placing the numerator of Equation (47) equal to zero, the relation between Q and A_c is as given by Equation (46*b*).

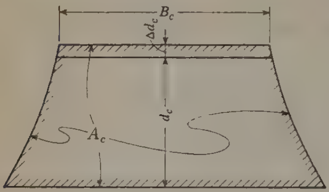

FIG. 4

As to the propriety of having a factor in the denominator which equals zero: As the numerator must equal zero to satisfy Equation (5) if the denominator is also zero it seems that the value of the fraction becomes $\frac{0}{0}$ which is generally indeterminate, although not always. In any event no factors in the denominator must, of necessity, be equal to zero, as the denominator may have almost any value and still satisfy the requirement for a maximum value of Q that $\frac{dQ}{dA_c} = 0$, if the numerator is zero.

Horace W. King, M. Am. Soc. C. E., describes the formula

$$\frac{a^3}{T} = \frac{Q^2}{g} \qquad (48)$$

as "a general criterion, applicable to all channels, indicating the relation between cross-section and discharge which must exist in order that flow may occur at critical depth."[10] Equation (48) resolves to

$$Q^2 = \frac{g \, A_c^3}{T_c} \qquad (49)$$

and is identical with Equation (46*b*) if $f'(A_c) = \frac{1}{T_c}$, T_c being the top width of the cross-section of the stream at the control section.

The writer has made a rough calculation of the area A_c, corresponding to a flow Q of 28.3 cu ft per sec, by using values given for d_c and B_c in Table 1(*a*). In this calculation it was assumed that the sides of the section in Fig. 2 were straight between the d_c-intervals. As the sides are curved, the value of 3.34 sq ft, obtained for A_c, is a little too large.

If this value is used in Equation (48), the resulting numerical expression is $\frac{(3.34)^3}{1.092} = \frac{(28.3)^2}{32.2}$, or $34.1 = 24.9$, which does not fulfil the requirements of the formula, as the two sides should be equal. If the precise value of A_c is found to be very close to 3.00 (which is probably the case), Equation (48) would check exactly and would be a proof of the correctness of Mr. Kennison's results.

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[10] "Handbook of Hydraulics," by Horace Williams King, McGraw-Hill Book Company, Inc., New York and London, Third Ed. (1939), p. 373.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

TENSION TESTS OF LARGE RIVETED JOINTS

Discussion

BY MESSRS. C. C. WINTER, W. M. WILSON, AND J. M. GARRELTS

C. C. WINTER,[20] ASSOC. M. AM. SOC. C. E. (by letter).[20a]—The writer proposes a few problems and tests, and suggests a few changes in specifications or in the interpretation of the present ones.

Rivets.—The rivet is the fundamental element of a riveted joint. The two main variables of rivets are the grip and diameter. It is suggested that there is, or should be, some relation between small pin and rivet design. Fig. 20 shows the relation between bearing, shear, and bending based on a simple case. If the bending is only half as much as assumed in Fig. 20, it still appears to be a very important factor. The tests reported in the paper indicate that a decrease in strength can be expected as the grip increases, but the tests have a very limited variation of grip. Fatigue tests at the University of Illinois support this conclusion. German fatigue tests have developed breaks in bending.

The writer suggests that the specifications on long rivets be made more conservative; for example, "Rivets (long rivets) subjected to calculated stress and having a grip in excess of two diameters shall be increased in number at least 16% per inch of grip." Tests to isolate this problem should be relatively simple.

In Fig. 20 it appears that, for thin plates, bearing is important. These tests were made on relatively thick plates so that bearing for thin plates was not revealed. There appears considerable evidence that the allowable bearing stress can be quite high. It is probable that the rivet heads confine and assist the joint by distributing the load over a larger area.

End Distance.—The tests reported in the paper did not indicate much need for greater end distances, but these tests were for relatively thick plates. One proposed specification calls for the end distance, or edge distance, to be such as to give an area equal to the shearing area of the rivet. It appears reason-

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NOTE.—This paper by Raymond E. Davis, and Glenn B. Woodruff, Members, Am. Soc. C. E., and Harmer E. Davis, Assoc. M. Am. Soc. C. E., was published in May, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1939, by Messrs. Charles F. Goodrich, Frederick P. Shearwood, and Jonathan Jones.

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[20a] Received by the Secretary August 14, 1939.

able to demand that this area be maintained between rivets to prevent tearing. Proposed: "The pitch should not be less than the end distance plus one-half the diameter of the hole."

Pitch.—In the "B" series, where full-row riveting was used, there is an indication that the maximum efficiency can be obtained when the pitch is two-thirds of the gage. This point can possibly be explained by the distribution of the Lueders lines. If the lines are at 45°, then with the pitch two-thirds of the gage, the maximum stress lines will fall between rivets and not through them. Photo-elastic studies should indicate the most efficient rivet pattern and the effect of pitch and gage on the stress in the plates.

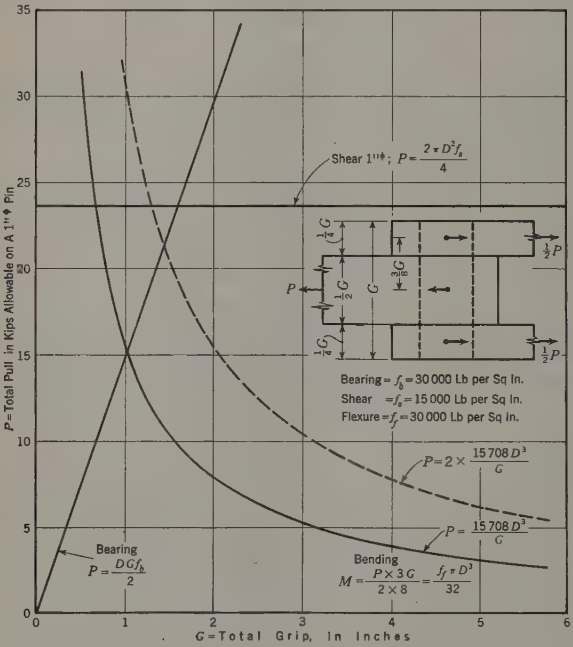

FIG. 20

Net Section.—These tests raised a large question about net section. Under the heading "Conclusions from Tests: Net Section" the following statement is made: "The experimental results bore no consistent relation to the specification requirements for determining net section."

The writer concurs that the tests do not agree with common practice and methods as shown in textbooks. However, net section, as used in the analysis of these tests and common practice, appears to be contrary to a strict interpretation of present specifications, which state that: "The net section shall be the least area which can be obtained by deducting from the gross sectional area, the area of holes cut by any straight or zigzag section across the member, etc." If the foregoing specification permits the protection of the net section by omitting rivets in the first row, then the specifications need revision.

The grooving effect is important when full-row riveting is used. Why not take advantage of this property and use the nominal diameter of the rivet when the holes are subpunched and reamed?

TABLE 26.—COMPARISON OF PROTECTED NET AREA AND LEAST NET AREA

Specimen (see Figs. 2, 3, and 4)	NET AREA, IN SQUARE INCHES		ULTIMATE STRENGTH, IN POUNDS PER SQUARE INCH		
	Protected area*	Least area†	Protected net area	Least net area	Coupon (mill)
CCC2-2-1	6.96	5.94	57.3	67.0	69.6
CCC2-2-2	6.93	5.92	56.3	65.9	69.6
CCC2-3-1	6.94	5.92	55.9	65.5	68.2
CCC2-3-2	6.95	5.94	56.3	65.8	68.2
CCC2-4-1	6.93	5.91	57.7	67.6	69.4
CCC2-4-2	6.87	5.87	58.7	68.7	69.4
CCC2-5-1	7.17	6.12	59.7	69.7	70.0
CCC2-5-2	7.18	6.13	57.1	66.8	70.0
CCC2-6-1	7.13	6.08	58.9	68.9	69.9
CCC2-6-2	7.16	6.10	58.2	68.3	69.9
CCC5-1	5.89	6.13	71.8	69.0	67.9
CCC5-2	5.91	6.14	69.9	67.2	67.9
CCC7-1	4.77	5.10	74.6	69.9	67.9
CCC7-2	4.79	5.13	75.2	70.1	67.9
DCC2-3-1	11.68	9.97	59.2	69.3	67.7
DCC2-3-2	11.71	9.99	60.0	70.3	67.7
DCC2-4-1	11.77	10.03	59.6	69.9	68.1
DCC2-4-2	11.76	10.03	59.4	69.6	68.1
DCC2-4-3	11.74	10.02	60.5	70.8	68.1
DCC2-4-4	11.76	10.03	61.0	71.5	68.1
DCC2-5-1	11.63	9.97	59.0	68.8	67.7
DCC2-5-2	11.65	9.95	59.7	69.8	67.7
DCC2-6-1	11.58	9.88	57.8	67.7	67.2
DCC2-6-2	11.60	9.90	59.0	69.2	67.2
DCC5-1	9.28	9.67	70.5	67.7	67.6
DCC5-2	9.29	9.67	72.2	69.4	67.6
DCC7-1	8.18	8.72	76.3	71.6	67.8
DCC7-2	8.18	8.72	76.0	71.3	67.8
FSCA-1	12.37	10.73	76.1	87.7	91.5
FSCA-2	12.46	10.81	76.8	88.5	91.5
FSCB-1	24.82	21.55	74.7	86.0	91.9
FSCB-2	24.86	21.58	74.2	85.6	91.9
FSCC-1	37.07	32.18	72.7	83.7	91.8
FSCC-2	36.90	32.02	71.2	82.1	91.8
FSCD-1	24.76	21.51	72.0	82.8	90.4
FSCD-2	24.89	21.60	73.0	84.0	90.4
FSCE-1	36.15	31.40	74.7	86.1	92.5
FSCE-2	36.93	32.05	72.4	83.4	92.5
FSCF-1	36.87	32.00	70.4	81.2	90.3
FSCF-2	37.13	32.22	72.9	84.0	90.3

* Protected net area is the first row net area as used in the tests.

† The least net area is based on the nominal diameter of the rivet, deducting one hole for each gage line.

Table 26 gives the ultimate strengths based on the nominal diameter of the rivet, one hole being deducted for each gage line, resulting in the least net section obtainable. It is found that the least net section is in fair agreement with the ultimate strengths based on mill coupons. A study of the data from the tests by E. L. Gayhart[6] at the Bureau of Standards also gives more reasonable results when based on the least net section.

One very desirable quality of any specification is a basis upon which the ultimate strength can be predicted. The present tests do not have large riveted joints with full-row riveting and it seems desirable to perform these tests before the specifications limit the riveted joints to 75% efficiency. A set of comparable butt-welded joints might also be worthy of consideration.

Proposed: "In calculating the required net section of riveted tension members, net sections shall be used in all cases; and in deducting rivet holes, the holes shall be taken as: (*a*) $\frac{1}{16}$ in. larger than the nominal diameter of the rivet for punched holes; (*b*) the nominal diameter of the rivet for reamed or drilled holes. In determining the net width, one hole shall be deducted for each gage line."

Transverse Shear.—In regard to the findings on transverse shear on the rivets, the tests indicate many coarse square fractures with very little reduction of area. If there is transverse shear on the rivets, there must be transverse tension in the main plates and compression in the cover plates. These stresses probably contributed to the curling and buckling of the plates. The combined stress appears to have caused the square breaks. The question might be put: "What is the effect on yield, ultimate strengths and reduction of area if the lateral contraction is restricted?"

The tests seem to indicate the desirability of using full-row riveting, short compact joints, and ductile materials.

It appears that a co-ordinated program of tests should be undertaken and then a uniform specification written.

W. M. WILSON,[21] M. AM. SOC. C. E. (by letter).[21a]—To the writer, the two most important conclusions from the tests given by the authors are:

(1) The ultimate strength of a large group of driven rivets equals, approximately, the sum of the strengths of the individual rivets; and,

(2) Omitting alternate rivets from the outside row of a group of rivets, or increasing the transverse distance between the rivets, does not add greatly to the effective net section of a riveted tension member. Conclusion (1) is reassuring and highly gratifying. Conclusion (2) has been indicated by other tests, and will be the subject of the present discussion.

The specifications most used in the design of American steel bridges contain rules for determining the effective net section of riveted tension members. These rules assign considerable importance to the relation between pitch and gage if some rivets are omitted from the outside row and do not recognize the effect of the transverse spacing of the rivets upon the unit strength of the section through the rivet holes. (The outside row of rivets of a joint corresponds to the outside row of a gusset plate in the case of a riveted tension member connected to a gusset plate.) This is true although there has been a considerable quantity of experimental data available for several years, indicating that the ratio of pitch to gage for staggered rivets is a minor factor and that the transverse spacing of the rivets is a major factor affecting the strength of riveted tension members. The tests reported in this paper are of value in that they add to the accumulating evidence that present specifications are in error in both of these respects.

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[6] "An Investigation of the Behavior and of the Ultimate Strength of Riveted Joints Under Load," by E. L. Gayhart, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 34, 1926, p. 55.

[21] Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

[21a] Received by the Secretary September 18, 1932.

The writer has searched American literature to obtain experimental data to be used in a study of the effective net section of riveted tension members such as are used for truss members of steel bridges. With a few exceptions, the tests included in this study comply with the following requirements: (1) The width of the specimens varied from 13.5 in. to 22.5 in., inclusive; (2) the strength of the plate material as given by tests of standard control specimens was reported; (3) the specimens failed in the plate; (4) the specimens had either $\frac{7}{8}$-in. or 1-in. rivets; and, (5) the joints were generally in accord with what is accepted as good design practice. However, there were the following exceptions to the requirements: One specimen was only 10 in. wide; four specimens failed in the rivets but at a plate stress that made the tests of value. Forty-nine tests reported by four investigators complied with these requirements, except as noted, and have been used in the study (they included eight tests made at the Watertown Arsenal;[22] twenty-one tests made for the Navy;[6] four tests by Mr. C. O. Harris;[23] and, sixteen tests made at the University of California and reported in the paper being discussed). In addition to the foregoing, a number of tests of specimens 9 in. wide, made by Mr. Harris,[23] have been used in the study of the effect of the ratio of pitch to gage upon the effective net section.

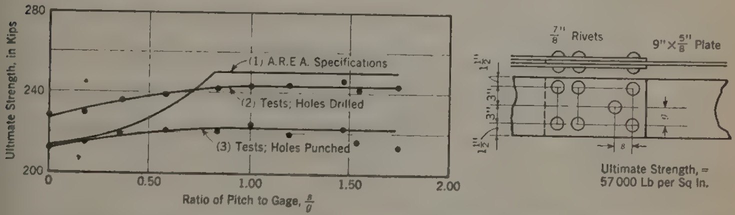

FIG. 21.—RELATION BETWEEN STAGGER OF RIVETS AND STRENGTH OF PLATE OF RIVETED JOINTS

Preliminary to the formulation of a rule for determining the effective net section of riveted tension members, the writer made one study to determine the effect of the ratio of pitch to gage on the strength of joints from which some rivets were omitted from the outer row, and he made a second study to determine the effect of the transverse distance between rivets upon the unit strength of the net section. These studies are presented herein.

The tests used in the study to determine the effect of the ratio of pitch to gage upon the strength of a joint from which rivets were omitted from the outer row include those made by Mr. Harris, some made for the U. S. Navy, and some made at the University of California. The tests made by Mr. Harris, and the details of the joint, are shown in Fig. 21. Two series of tests of riveted joints are reported: For one the holes in the plates were punched (Curve (3)), and for the other the holes were drilled (Curve (2)). A third series of tests was made by Mr. Harris for which the specimens were plates without joints, but with open holes of the same pattern as the rivets of the specimens with riveted joints. The results of these latter tests are shown by Curve (1) in Fig. 21. Tests were made in duplicate for each series and the small circles represent the average of the results of tests on duplicate specimens. The ordinates of the diagrams represent the ultimate load and the abscissas represent the ratio of the pitch to the gage. The strength of the plate material was determined by tests of standard control specimens for all of Mr. Harris' tests, and the ultimate strength for all specimens has been adjusted to a common plate strength of 57 000 lb per sq in. It is to be noted that the plates with open holes tested by Mr. Harris were stronger than the plates connected with rivets. The relation between the strength and the stagger, as computed by the American Railway Engineering Association specifications, is given in Curve (1), Fig. 21.

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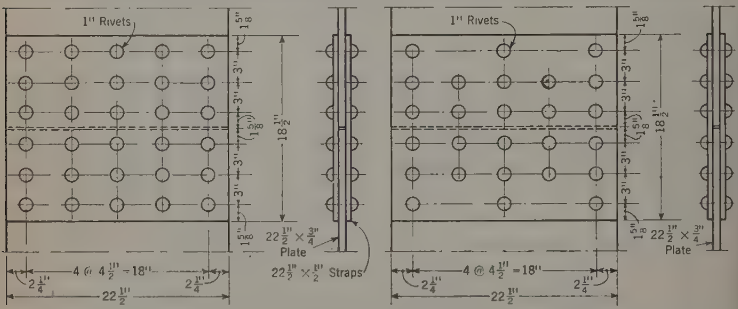
[22] "Tests of Metals," 1896, p. 247.

[6] "An Investigation of the Behavior and of the Ultimate Strength of Riveted Joints Under Load," by E. L. Gayhart, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 34, 1926, p. 55.

[23] "Effective Net Section of a Riveted Joint," by C. O. Harris, thesis presented to the University of Illinois in 1930 in partial fulfillment of the requirements for the degree of Master of Science.

Tests made for the U. S. Navy by E. L. Gayhart,[6] and used in this study, include six each with the rivet patterns shown in Table 27. Each group of

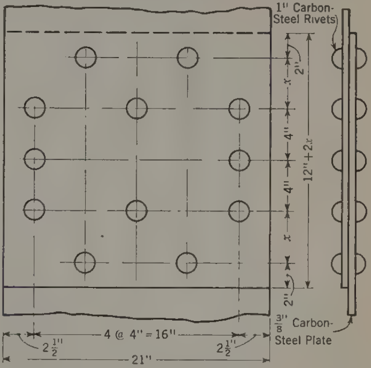
TABLE 27.—INFLUENCE OF RELATION OF PITCH TO GAGE UPON EFFECTIVE NET SECTION OF RIVETED TENSION MEMBER; TESTS FOR NAVY



TYPE *B* TYPE *D*

Item No.	Description	TYPE *B*		TYPE *D*	
		Medium steel	High-test steel	Medium steel	High-test steel
1	Maximum load, in pounds	824 000	1 089 000	853 000	1 053 000
2	Stress on gross section, in pounds per square inch	50 200	65 960	52 690	64 010
3	Strength of control specimen, in pounds per square inch	59 760	80 740	64 975	80 320
4	Effective net area, by test, in square inches	13.79	13.49	13.13	13.11
5	Net Section by American Railway Engineering Association specifications, in square inches	12.31	12.38	13.58	13.80

six specimens included three with carbon-steel plates and three with low-alloy plates. For the specimens with carbon-steel plates, the efficiency of the joints, the average of three tests in each instance, was 0.84 for Type B and 0.81 for Type D. For the specimens with low-alloy plates the corresponding efficiencies were 0.82 and 0.80. That is, changing the pitch from 0 to $\frac{2}{3}$ of the gage decreased the efficiency of the joint only about 3 per cent. (The term efficiency as used herein is the ratio of the ultimate strength as determined by test to the product of the gross area of the section and the unit strength of the control specimen.)



SPECIMEN *CCC2*

FIG. 22

Two series of tests made at the University of California, and reported by the authors (Specimens No. *CCC*2 and *DCC*2, Fig. 2 and Table 5), had the rivet pattern shown in Fig. 22. The plate thickness was $\frac{3}{8}$ in. for Specimen *CCC*2 and $\frac{5}{8}$ in. for Specimen *DCC*2. The pitch, x, had values of 2 in., 3 in., 4 in., 5 in., and 6 in. for *CCC*2, and 3 in., 5 in., and 6 in. for *DCC*2. The values of the efficiency of the joints for the various values of x were as shown in Table 28. It is to be noted that varying x produced no large or consistent change in the efficiency of the joints.

TABLE 28.—INFLUENCE OF RELATION OF PITCH TO GAGE UPON EFFECTIVE NET SECTION OF RIVETED TENSION MEMBERS. TESTS AT UNIVERSITY OF CALIFORNIA

Item No.	Description	SPECIMEN *CCC2*					SPECIMEN *DCC2*		
		$x = 2$ inches	$x = 3$ inches	$x = 4$ inches	$x = 5$ inches	$x = 6$ inches	$x = 3$ inches	$x = 5$ inches	$x = 6$ inches
1	Ultimate load, in thousands of pounds per square inch.	399.0	388.0	400.0	428.0	420.0	691.0	686.0	669.0
2	Stress on gross section, in pounds per square inch . . .	51.2	49.9	51.5	53.3	52.6	52.9	52.7	51.6
3	Strength of control specimen, in pounds per square inch.	67.7	67.0	66.7	67.5	68.5	65.8	65.6	65.1
4	Effective net section, by test, in square inches.	5.89	5.79	6.00	6.34	6.13	10.50	10.46	10.28
5	Net section by American Railway Engineering Association specifications, in square inches.	6.08	6.52	6.93	7.17	7.13	10.97	11.62	11.57
6	Efficiency.	0.756	0.745	0.772	0.790	0.768	0.804	0.803	0.793

The results of the three independent investigations,[6, 23] agree in one important respect: The strength of riveted joints, with some rivets omitted from the outer row, is not greatly affected by the ratio of the pitch to the gage.

In the study to determine the effect of the transverse distance between rivets on the unit strength of the net section of the plates, only two types of joints were included: (1) Joints with no rivets omitted from the outer row; and (2), joints with some rivets omitted from the outer row but only those which had a pitch equal to, or greater than, the gage. They included tests from all four investigations. The relation between the ratio, $\frac{\text{Strength of net section}}{\text{Strength of control specimen}}$, and the transverse distance between rivets is shown by the small circles of Fig. 23.

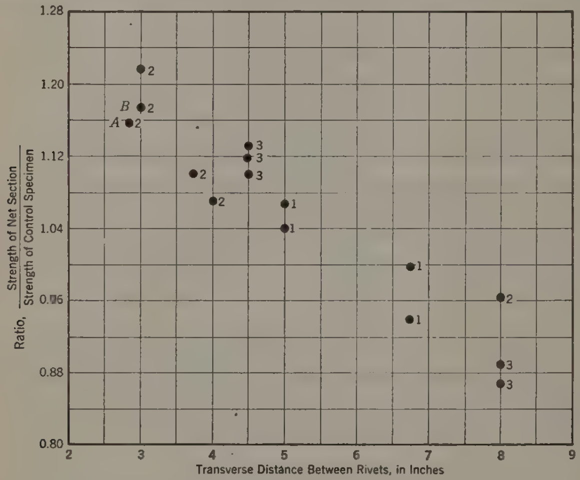


FIG. 23

The numeral adjacent to each circle indicates the number of tests of identical specimens averaged. The study includes twenty-eight tests and a variation in gage from $2\frac{7}{8}$ in. to 8 in. The points marked *A* and *B* are for specimens that failed in the rivets. It is apparent from Fig. 23 that the unit strength of the net section of a riveted tension member is greatly affected by the transverse distance between the rivets.

In view of the discussion contained in the previous paragraphs, it appears that any rule for determining the effective net section of a riveted tension member should take into consideration the net section through the outer row of rivets and also the transverse distance between rivets. Furthermore, it would appear unnecessary to consider the effect of minor variations in the ratio of the pitch to the gage for joints with rivets omitted from the outer row. With these ideas in mind the following empirical rule for determining the effective net section has been developed.

The effective net width (W_n) of a riveted tension member is given by the equation

$$W_n = (W_g - N\,D)\left(1 + \frac{D}{G}\right)0.85 \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

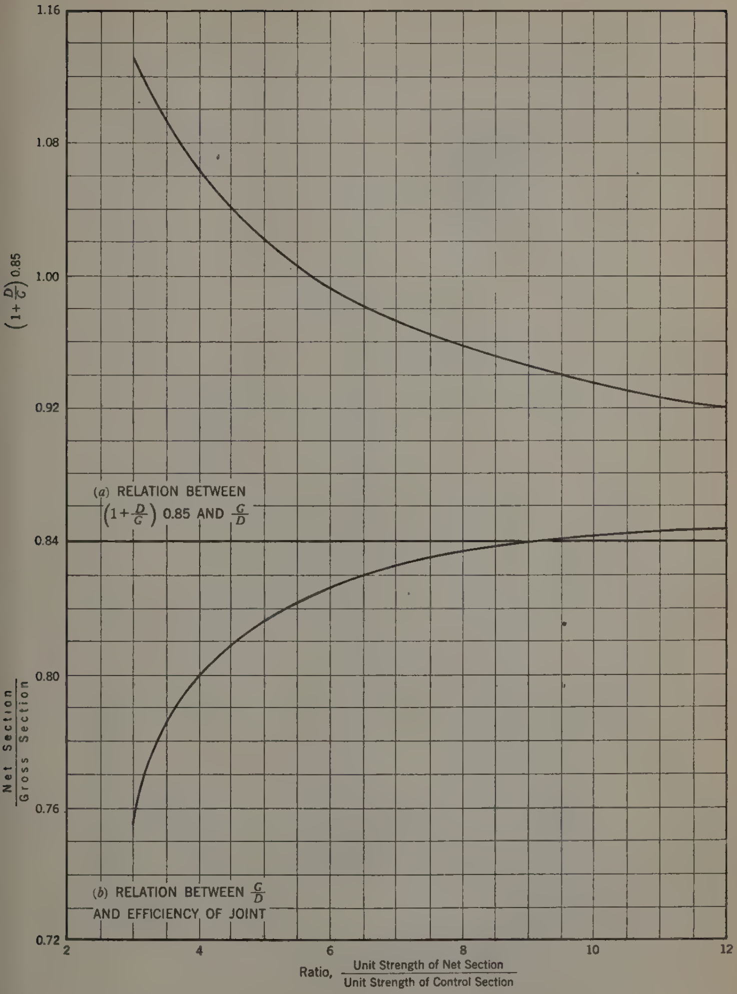

1.16
1.08
1.00
0.92
0.84
0.80
0.76
0.72
$\left(1+\frac{D}{G}\right)0.85$
$\frac{\text{Net Section}}{\text{Gross Section}}$
(a) RELATION BETWEEN $\left(1+\frac{D}{G}\right)$ 0.85 AND $\frac{G}{D}$
(b) RELATION BETWEEN $\frac{G}{D}$ AND EFFICIENCY OF JOINT
2
4
6
8
10
12
Ratio, $\frac{\text{Unit Strength of Net Section}}{\text{Unit Strength of Control Section}}$

FIG. 24

in which W_g = gross width of the member; D = nominal diameter of the rivet plus $\frac{1}{8}$ in.; N = number of rivet holes to be deducted; and, G = transverse distance between rivets, or two times the edge distance, whichever is the greater.

If rivets are omitted from the outer row, N and G shall be determined by the rivets in the outer row provided the pitch equals or exceeds 0.80 of the gage; otherwise the intervening rivets in the second row and all of the rivets in the outer row will be considered in determining N and G.

Equation (4) is composed of two parts: One is the actual net width, and the other is a factor to compensate for the effect of the transverse distance between rivets upon the ratio, $\frac{\text{Unit strength of net section}}{\text{Unit strength of control specimen}}$. Values of this latter factor for various values of $\frac{G}{D}$ are shown in Fig. 24.

The proposed rule for determining the effective net section of riveted tension members has been applied to the forty-nine tests of the four investigations cited[6, 22, 23] at the beginning of this discussion and the results are presented in Table 29. In this table, Column (5) contains values of G determined in the manner described in the preceding paragraphs. Column (6) gives the efficiency of the joint, the efficiency being the ultimate load, as determined by test, divided by the product of the gross area and the unit strength of the control specimen. Column (7) gives the effective net section as given by test, this being the ultimate strength, as given by the test, divided by the unit strength of the control specimen. Columns (8) and (9) give the computed values of the net section, the former being by the A. R. E. A. specifications for railway bridges and the latter by the proposed specifications. Columns (10) and (11) give the ratios of the computed to the test values of the net section, the former being for the A. R. E. A. specifications and the latter for the proposed specifications. A value greater than unity indicates that the computed value exceeds the value determined by test. A perusal of the values in Columns (10) and (11) reveals the agreement between the computed values and the values determined from tests.

It is of interest to note that the largest value of R given in Column (10) is 1.149, in Line 21, and the smallest value is 0.820, in Line 15, indicating that the A. R. E. A. specifications gave a value for the net section 15% too great for one specimen and 18% too small for another specimen. The largest value of R_1 given in Column (11) is 1.061, in Line 21, and the smallest value is 0.928, in Line 15, indicating that the maximum difference between the two values, one given by the proposed specifications and the other by the tests, is only 7 per cent. It is to be noted further that, in general, the proposed specifications give a value more nearly equal to the value given by tests than do the A. R. E. A. specifications. Moreover, the proposed specifications penalize the bad practice of using an extremely large transverse spacing of the rivets and large edge distances in the outside row of rivets, expedients which, according to A. R. E. A. specifications, result in a high efficiency but which, according to tests, resulted in little, if any, increase in efficiency.

In the proposed rule, G is defined as the transverse spacing, or two times the edge distance, whichever is the greater. If the transverse spacing varies, the widest space, or two times the edge distance, is to be used.

TABLE 29.—EFFECTIVE NET SECTION OF RIVETED TENSION MEMBERS; CARBON-STEEL RIVETS AND CARBON-STEEL PLATES

Line No.	No. of tests averaged	NOMINAL DIMENSIONS, IN INCHES		Value of G, in inches	Efficiency of joint	NET SECTION, IN SQUARE INCHES BY:			$R = (8) \div (7)$	$R_1 = (9) \div (7)$
		Thickness of plate	Diameter of rivet			Tests	American Railway Engineering Association specifications	Proposed specifications		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
				(*a*) WATERTOWN ARSENAL TESTS						
1	1	0.50	0.875	6.75	0.802	5.40	5.73	5.10	1.061	0.945
2	1	0.50	0.875	5.00	0.835	4.10	3.93	4.01	0.959	0.977
3	1	0.50	0.875	5.00	0.855	6.03	5.64	5.75	0.935	0.954
4	1	0.50	0.875	6.75	0.853	8.47	8.46	8.24	1.000	0.973
5	1	0.50	0.875	6.75	0.825	8.32	8.93	8.38	1.073	1.007
6	1	0.50	0.875	10.00	0.785	8.41	9.33	8.51	1.109	1.012
7	2	0.50	0.875	5.00	0.873	6.48	6.38	6.06	0.985	0.934
				(*b*) U. S. NAVY TESTS						
8	3	0.75	1.000	4.50	0.850	14.04	12.39	13.16	0.883	0.937
9	3	0.75	1.000	4.50	0.840	13.79	12.31	13.08	0.893	0.948
10	3	0.75	1.000	4.50	0.823	13.29	12.11	12.86	0.911	0.968
11	3	0.75	1.000	4.50	0.811	13.13	13.58	12.90	1.034	0.982
12	3	0.75	1.000	4.50	0.809	13.01	13.47	12.81	1.035	0.984
13	3	0.75	1.000	4.50	0.851	13.72	13.54	12.86	0.987	0.937
14	3	0.75	1.000	4.50	0.843	13.86	13.79	13.09	0.995	0.945
				(*c*) TESTS BY C. O. HARRIS						
15	2	0.625	0.875	3.00	0.812	12.20	10.00	11.32	0.820	0.928
16	2	0.625	0.875	8.00	0.813	9.64	10.00	9.55	1.037	0.990
				(*d*) UNIVERSITY OF CALIFORNIA TESTS						
17	2	0.375	1.000	3.00	0.708	5.52	4.78	5.58	0.866	1.010
18	2	0.625	1.000	3.00	0.735	10.22	8.70	10.16	0.851	0.994
19	2	0.375	1.000	5.00	0.783	6.31	5.90	6.14	0.935	0.973
20	2	0.625	1.000	5.50	0.797	10.20	9.28	9.49	0.910	0.931
21	3	0.375	1.000	13.00	0.777	6.16	7.08	6.54	1.149	1.061
22	1	0.375	1.000	5.00	0.756	5.89	6.08	5.93	1.032	1.007
23	1	0.375	1.000	5.00	0.745	5.79	6.53	5.92	1.128	1.023
24	2	0.625	1.000	13.00	0.798	10.37	11.60	10.71	1.119	1.035
25	1	0.625	1.000	5.00	0.804	10.50	10.96	9.94	1.044	0.947

If all of the transverse spaces are equal and if the edge distance is one-half of the transverse spacing, the net section can be expressed as a fraction of the gross area and the value of the fraction depends only upon the ratio of G to D. The relation between the ratio $\frac{\text{Effective net section}}{\text{Gross section}}$ and the ratio $\frac{G}{D}$ is given in Fig. 24(*b*). It is apparent from this diagram that the efficiency of a joint does not increase appreciably with an increase in the transverse spacing beyond a value of $G = 6\ D$.

The authors have drawn the conclusion that the efficiency of a riveted tension member does not greatly exceed 75%, irrespective of the transverse spacing of the rivets. If a different rivet pattern had been used for the joints with a large transverse spacing, it is believed that a different conclusion would have resulted. All of the specimens with a large transverse spacing that were tested by the authors had no rivets at the ends of the outside row, and the edge distance was abnormally large for the weakest section of the main plate. Thus, for Specimens *CCC*2 and *DCC*2, shown in Fig. 22, the edge distance for the outside row of rivets is 6.5 in. Geometrically, this is equivalent to a transverse rivet spacing of 13 in., whereas, for the interior rivets of the outside row, the spacing is only 8 in. Moreover, the authors report in another part of the paper that, beyond the yield point, the unit longitudinal strain in a wide plate is greater over the middle than it is at the edges. This being true, the harmful effect of placing the rivets on the weak section near the mid-width accentuates the harmful effects of a wide edge distance. This statement is supported by the tests reported in Table 29, which are discussed in the following paragraphs.

The only specimens for which the value of the effective net section determined by the proposed specification exceeds the value given by the tests is for those tests reported in Lines 5, 6, 17, 21, 22, 23, and 24, Table 29.

The details of the specimen used in the test reported in Line 5 are shown in Table 30 (Specimen 8510). The edge distance is $3\frac{3}{8}$ in. The proposed specifications give a value for the effective net section 0.70% greater than the value given by the test.

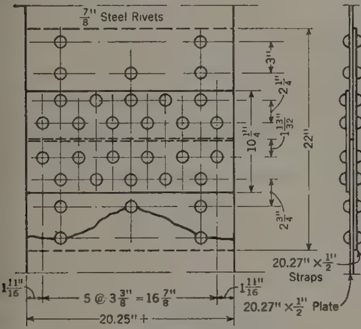
The details of the specimen used in the test reported in Line 6 are also given in Table 30 (Specimen 9375). The edge distance is 5 in. The proposed specifications give a value for the effective net section 1.2% greater than the value given by the tests.

The specimens used in the tests reported in Line 17, Table 29, had a close spacing and a small edge distance; but the fact that the computed value of the effective net section exceeded by 1.0% the value given by the tests is accounted for by the fact that the failure of the specimens was due to the shearing of the rivets and not by a failure of the plate. All other specimens for which the computed value of the effective net section exceeded the test value had the same rivet pattern as the specimens of Fig. 22. That is, the few instances in which the computed value of the effective net section exceeded the test value can be accounted for by the fact that the edge distance was large for the outer row of rivets.

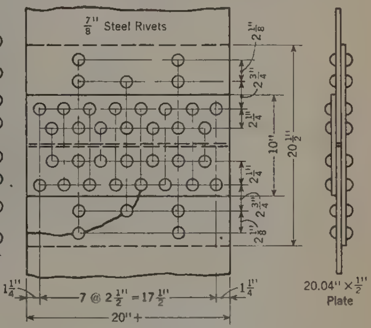
The specimens shown in Table 31 have practically the same transverse spacing of rivets, but Specimens *DCC*2 have two rivets near the middle of the outer row, whereas Specimens of Type *F* have one rivet at the center and one at each end of the outer row.[6] The computed and test values of the efficiency are equal to each other and equal to 0.80 for Specimens *DCC*2, whereas, for Specimens of Type *F*, the computed efficiency is 0.80 and the value determined from the test is 0.85, the values being the averages from three tests in each instance. That is, although the computed value of the effective net section for Specimens *DCC*2 is reduced because of the large edge distance, the computed value is still greater, relative to the test value for these specimens, than it is for the specimens of Type *F*, which have a small edge distance.

The fact should be noted, however, that the specimens used for the tests reported in Lines 1, 2, 3, and 4, Table 31, had edge distances equal to one-half the transverse spacing and yet they developed an effective net width somewhat greater than the computed value.

TABLE 30.—STRENGTH OF RIVETED TENSION MEMBERS FOR WHICH THE EDGE DISTANCE OF THE RIVETS IN THE OUTSIDE ROW IS EXCESSIVE


$\frac{7}{8}$" Steel Rivets
3"
$2\frac{1}{4}$"
$10\frac{1}{4}$"
$1\frac{13}{32}$"
22"
$2\frac{3}{4}$"
20.27" × $\frac{1}{2}$" Straps
$1\frac{11}{16}$"
5 @ $3\frac{3}{8}$" = $16\frac{7}{8}$"
$1\frac{11}{16}$"
20.27" × $\frac{1}{2}$" Plate
20.25" +

SPECIMEN 8510
Joint Failed in Plate, Holes Drilled



SPECIMEN 9375
Holes Drilled

Item No.	Description	Specimen 8510	Specimen 9375
1	Maximum load, in pounds	503 970	502 200
2	Stress on gross section, in pounds per square inch	49 960	46 900
3	Strength of control specimen, in pounds per square inch	60 550	59 700
4	Effective net section, by test, in square inches	8.32	8.41
5	Net section by American Railway Engineering Association specifications	8.93	9.33

As a result of the analysis of the data contained in Table 29, the following rules for the design of riveted tension members are submitted for the consideration of the profession:

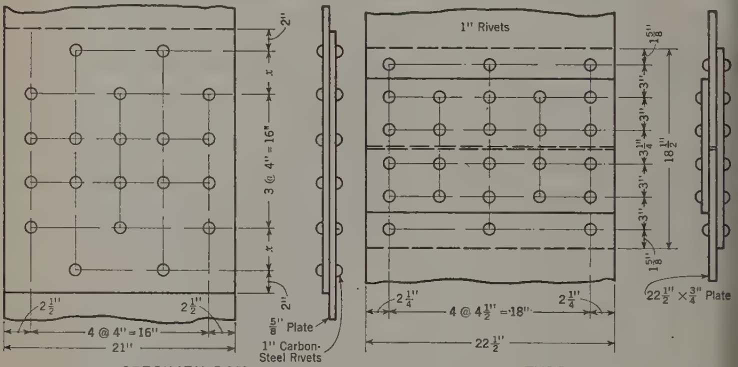
Rule 1. Effective net width $= (W_g - N\,D)\left(1 + \frac{D}{G}\right)0.85$.

Rule 2. The edge distance shall not exceed 2.5 *D* for the outer row of rivets.

In the foregoing equation (Rule 1): W_g = gross width of the member; N = number of rivet holes to be deducted; D = nominal diameter of the rivets plus $\frac{1}{8}$ in.; and G = transverse distance between rivets or two times the edge distance, whichever is the greater.

For joints with rivets omitted from the outer row, only rivets in the outer row are to be considered in determining N and G, provided the pitch equals or exceeds 0.80 of the gage; otherwise all rivets in the outer row and the intervening rivets in the second row should be considered in determining N and G.

TABLE 31.—INFLUENCE OF EDGE DISTANCE OF RIVETS IN OUTSIDE ROW UPON STRENGTH OF RIVETED TENSION MEMBERS



SPECIMEN *DCC2*
Carbon-Steel Plates Holes Sub-Punched and Reamed

TYPE *F*
Holes Drilled

Item No.	Description	SPECIMEN *DCC2*			SPECIMEN, TYPE *F*	
		$x = 3$ inches	$x = 5$ inches	$x = 6$ inches	Medium steel	High-test steel
1	Ultimate load, in kips	691.0	686.0	669.0	902.0	1 113.0
2	Stress on gross section, in kips per square inch	52.9	52.7	51.6	55.9	66.2
3	Strength of control specimen, in kips per square inch	65.8	65.6	65.1	65.7	81.6
4	Effective net section, by test, in square inches	10.50	10.46	10.28	13.72	13.64
5	Net section by American Railway Engineering Association specifications, in square inches	10.97	11.62	11.57	13.54	14.11

In the preceding discussion it has been assumed that the weakest section of a riveted tension member is through the outside row of rivets of the gusset plate at the ends of the member. The writer knows of no published reports of tests of riveted tension members with stitch rivets, but the tests of plates with holes by Mr. Harris, reported in Fig. 21, indicate that an open hole does not weaken a plate as much as a rivet transmitting stress. It is not reasonable to suppose that a stitch rivet weakens a plate more than an open hole. It would seem reasonable, therefore, that the foregoing rules of design, which are applicable where the rivets transmit stress, give conservative results where the rivets do not transmit stress but only serve as stitch rivets. The only probable exception to this statement is where stitch rivets are used to connect two parallel pieces of steel having widely differing stress-strain diagrams. Under this latter condition one piece, because of its lower ductility, might break before the other had been stressed to its ultimate strength. This condition is not likely to occur in a member fabricated of a carbon steel of structural grade because the part of the stress-strain diagram near the maximum stress is nearly horizontal for a considerable length. However, the behavior, under similar conditions, of a member fabricated from parallel pieces made of a steel which is not uniform in its physical properties is somewhat uncertain.

J. M. Garrelts,[24] Assoc. M. Am. Soc. C. E. (by letter).[24a]—The engineering profession is indebted to the engineers of the San Francisco-Oakland Bay Bridge who provided for this important series of tests, and to those who were responsible for making the tests and analyzing the results. As has been advocated by some consulting engineers, a series of well-planned tests, made in connection with the important structures which are built, would soon yield valuable information concerning the behavior of structures.

The fact that under different stresses "the plates in a riveted joint tend to elongate by unequal increments" is important and should be used in the analysis of many structural problems as well as in that of the riveted joint. As allowable unit stresses increase, structural design must be placed on a more scientific basis. The structural designer must be aware of the many indeterminate factors that affect the action of a structure and must have some knowledge of the influence of those factors.

In order to determine the effective net section and the effect of end pitch, additional data are needed concerning the behavior of plates with open holes under stress. The lines of fracture for the specimens shown in Figs. 6 and 7 all appear to originate in the regions of high stress concentration. When a condition of plastic yielding is reached, a redistribution of stress takes place, but initial fracture may occur in the region where yielding first occurs. If stress concentration at the edges of a hole has any bearing on the initial fracture, t is quite probable that the present ideas concerning net section need to be revised. The writer is of the opinion that, by additional study and tests, a rational analysis may result. In the meantime, the authors' recommendations appear to represent a reasonable procedure.

In view of the uncertain action of the riveted joint in Stage II, the authors' recommendations for assuming equal shear per rivet seem fully justified and no additional refinement is likely to result from further tests.

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[24] Associate Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[24a] Received by the Secretary September 18, 1939.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

LARGE CORE DRILLS AID CONSTRUCTION AT CHICKAMAUGA DAM

Discussion

By J. B. Newsom, Esq.

J. B. Newsom,[2] Esq. (by letter).[2a]—Most engineers do not understand that drills which will bore holes large enough for a man to enter can be used in any ground, from the loose water-bearing ground containing some boulders described in the paper, to the hardest rocks known to the mining industry. In the work described, it was necessary to use casing. Diameter was lost every time it was placed, so that a hole which started 4 ft in diameter might be 3 ft in diameter at the bottom. In boring a hole 1 125 ft deep in California in 1935,[3] the hole went through heavy ground and gouge which had to be supported without losing size. This was accomplished satisfactorily by using an expanding steel form, behind which quick setting concrete was packed. These large drills are capable of higher development and have a much wider field of possible usefulness than is generally recognized.

Note.—This paper by James S. Lewis, Jr., Assoc. M. Am. Soc. C. E., was published in June, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[2] With Idaho Maryland Mines Corp., San Francisco, Calif.

[2a] Received by the Secretary August 14, 1939.

[3] "Shaft Sinking with a Shot Drill, Idaho Maryland Mine, Grass Valley, Calif.," by J. B. Newsom and C. F. Jackson, U. S. Bureau of Mines *Information Circular 6923.*

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

STRESS DISTRIBUTION AROUND A TUNNEL

Discussion

BY MESSRS. W. O. RICHMOND, AND JACOB FELD

W. O. RICHMOND,[21] ESQ. (by letter).[21a]—The stress distribution due to gravity in a semi-infinite elastic body pierced by a circular hole is an interesting problem and the author's treatment of it is a complete one. The application of the results in practise depend, of course, upon the accuracy with which the assumptions are fulfilled. If the shape of the tunnel is other than circular (as it often is), a mathematical solution is practically impossible and recourse must be had to experimental methods.

An interesting experimental approach to this problem was developed by Mr. S. Yamaguti,[22] whose method consisted of casting a flat plate of agar-agar in a horizontal position. A network of lines was placed on the plate, and a hole was cut, of a shape appropriately representing the tunnel profile. When the model was placed in a vertical position, the strain in the material due to its own weight could be measured from the distortion of the originally parallel lines, and hence the stresses could be determined. For a round tunnel profile Mr. Yamaguti obtained good agreement between his mathematical solution and the results of his model test.

Mr. Yamaguti's method of test reproduces the author's Case II with the appropriate value of Poisson's ratio for agar-agar. The author's Case I could also be reproduced by this method. It would only be necessary to cast the agar-agar plate in a vertical position, so that the stress distribution in the unpierced plate would approximate a hydrostatic pressure.

A second experimental approach to the problem is by the method of photoelasticity. If the tunnel profile is some distance below the surface, the stress distribution may be obtained approximately by the method of superposition, as stated by the author. Assuming that the original stress distribution is as

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NOTE.—This paper by Raymond D. Mindlin, Assoc. M. Am. Soc. C. E., was published in April, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion on the paper.

[21] Asst. Prof. of Mech. Eng., Univ. of British Columbia, Vancouver, Canada.

[21a] Received by the Secretary June 15, 1939.

[22] "On the Investigation of the Stress Distribution in a Tunnel with Agar-agar Model Experiments," by S. Yamaguti, *Proceedings*, Third International Cong. for Applied Mechanics, Vol. II, 1930, pp. 150–154.

represented by Case II, for instance, the boundary stress around the profile is just determined for a uniform compression in a vertical direction of magnitude.

$$\sigma_y = - w\,c \qquad (69)$$

Then the boundary stress is determined, for a stress at right angles, of magnitude

$$\sigma_x = - \frac{\nu}{1-2\,\nu}\,w\,c \qquad (70)$$

in which the symbols used are those of the author in Fig. 2, and c is the distance of the hole from the free surface. The stress on the boundary is then obtained by adding the two boundary stress distributions. For a non-circular profile the boundary stresses are most easily determined by photo-elastic tests.

This procedure is sufficiently accurate if the profile is some distance below the surface. For the case shown in Fig. 4(*b*) (that is, for a value of Poisson's ratio of ½ and $\alpha_1 = 2.0$), this approximate method gives a uniform compressive stress around the boundary of the tunnel of $-3.76\,w\,D$, in which w = density of the material, and D = diameter of the circular profile. The maximum stress by the author's computation is $-4.26\,w\,D$. The error in the approximate method is thus slightly more than 10%, which is perhaps accurate enough for practical purposes. Thus, the conclusion may be drawn that if the tunnel is more than four radii, or two diameters from the surface, the approximate solution obtained by superposition is sufficiently accurate.

A few years ago the writer (working under the direction of A. Nadai at the Westinghouse Research Laboratories) made some photo-elastic tests on holes of tunnel profile at the suggestion of F. C. Carstarphen, M. Am. Soc. C. E. The profiles tested were squares with various fillet radii at the corners; the ratio of fillet radius, r, to side of square, D, had the values $\frac{r}{D} = \frac{1}{3}, \frac{1}{6}$, and $\frac{1}{12}$. The boundary stress distribution was obtained photo-elastically when the bakelite plate which contained the holes was subjected to a pure tension parallel to an axis of symmetry perpendicular to the sides of the holes. The stress concentration factors (that is, the ratio of the maximum stress on the boundary of the hole to the stress in the plate a long way from the hole) from these tests are shown in Table 6. For the sake of completeness, the case of the circular hole in a tensile stress field, for which $\frac{r}{D} = \frac{1}{2}$, is included, the results for which can be

TABLE 6.—STRESS CONCENTRATION FACTORS

Item No.	Description	RATIO OF FILLET RADIUS, r, TO THE SIDE OF THE SQUARE, D			
		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$
1	Stress concentration factor for tension or compression	3	2.6	2.5	2.6
2	Maximum compressive stress on boundary of tunnel profile (Case II, $\nu = \frac{1}{4}$)	2.7	2.4	2.4	2.5

obtained by computation.[23] It is interesting to note that a minimum stress concentration factor seems to result for a value of $\frac{r}{D}$ of about $\frac{1}{6}$. The stress concentration factor for a square hole with sharp corners would be theoretically infinite.

The stress distribution around the boundary of tunnels of similar cross-section was then computed from the boundary stresses determined by the photo-elastic tests. The case solved by the writer was the author's Case II, in which the principal stresses before the tunnel existed were given by Equations (69) and (70). The results obtained by this procedure for a value of Poisson's ratio of $\frac{1}{4}$ are shown in Item 2, Table 6. These results should be sufficiently accurate, provided that the tunnel is at least two diameters below the surface, that the material in which the tunnel is driven behaves elastically, and that the original principal stresses are as assumed.

Jacob Feld,[24] M. Am. Soc. C. E. (by letter).[24a]—For the condition explained in this paper—namely, a circular tunnel in a uniform elastic material—Mr. Mindlin's solution is the most complete available. For use in practical cases, however, the following items must be taken into account:

I.—The internal stress in the rock in its natural condition, which stress may be relieved in the local vicinity of the tunnel by the removal of the tunnel rock.

II.—The percentage of error resulting from the use of the method and formulas in this paper when rock conditions cannot be assumed as isotropic. It is seldom that a tunnel is driven through a uniform granite, which is probably the nearest approach to the material assumed in this paper. The more usual cases are tunnels driven through laminar rock, such as shales, schists, or limestones. It is extremely doubtful whether the stress determinations resulting from the formulas in this paper can be applied to a case where a blocky limestone is encountered, or, to take a different example, where schist in thin layers, lying edgewise to the face of the tunnel, is encountered.

III.—The effect on stresses around the tunnel if temporary bracing is wedged against the exposed rock during driving, or the effect on the rock stresses as the tunnel is lined with concrete and grouted under pressure; also the effect of various grouting pressures in balancing or over-balancing rock stresses.

In taking the foregoing suggestions into consideration, it might be pertinent to consider the assumption made under the heading "Statement of the Problem" that, at the circular boundary, the normal stress and the shearing stress are equal to zero. Such an assumption is not correct when rock is encountered which requires bracing to prevent movement. It seems to the writer that in the three cases mentioned as covering the initial states of the rock, although they cover a great range of possibilities, such range is chiefly in the variation of the Poisson ratio in Case II. Case I is that of hydrostatic pressure which may be applicable at tremendous distances below the earth's surface in rocks that are under such pressure that they are practically liquid, and Case III would only apply where a tunnel is driven through blocks of rock with open vertical seams, which case could not be considered isotropic. With reference to Case II and its dependence upon the Poisson ratio of the material, it might well be emphasized that such ratio may vary for rocks, depending upon the amount of the stress. However, as stated in Mr. Mindlin's paper, such variation has little effect on the stresses themselves, except when the tunnel is unusually close to the open surface. It is for such a condition (namely, shallow tunnels) that a careful stress analysis is most necessary.

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[23] "Theory of Elasticity," by S. Timoshenko, 1934, p. 75.

[24] Cons. Engr., New York, N. Y.

[24a] Received by the Secretary August 1, 1939.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

DESIGN OF A HIGH-HEAD SIPHON SPILLWAY

Discussion

BY MESSRS. B. E. TORPEN, G. E. HYDE, AND R. B. COCHRANE

B. E. TORPEN,[10] M. AM. SOC. C. E., G. E. HYDE,[11] JUN. AM. SOC. C. E., and R. B. COCHRANE,[12] ESQ. (by letter).[12a]—The approach to the problem of siphon spillway design presented in this paper may prove meritorious. Equation (4), ased on the assumption that free vortex flow prevails in the bend of the throat ection of a siphon spillway is a simplifying step, and results in doing away with he necessity of selecting a "coefficient of ignorance" masked under the name of Flow Coefficient. Therefore, his attempt to rationalize the common procedure f design for throat sections of siphon spillways is worthy of commendation.

Vortex Flow Assumption.—Certain questions and doubts arise, however, oncerning the soundness of the author's assumption of vortex flow in siphon hroat sections, and concerning the practical application of his formula in the lesign of siphon spillways. In an attempt to answer these questions and to liminate these doubts, the velocity distributions, as they actually exist in the hroat sections of three different siphon spillways[13] were compared, respectively, o the velocity distributions as computed from Mr. Rock's formula (see Equaion (10)):

$$V = \frac{Q}{r \log_e \frac{r_s}{r_c}} \qquad \qquad (11)$$

he numerical calculations involved in this comparison are given in Table 3 nd the results are portrayed graphically in Fig. 4. Table 3 first lists the ctual operating conditions of the three selected siphons. These data were

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NOTE.—This paper by Elmer Rock, Jun. Am. Soc. C. E., was published in April, 1939, *Proceedings*. iscussion on this paper has appeared in *Proceedings*, as follows: June, 1939, by J. C. Stevens, M. Am. oc. C. E.

[10] Prin. Engr., Corps of Engrs., U. S. A., Bonneville, Ore.

[11] U. S. Engr. Office, Bonneville, Ore.

[12] Assoc. Engr., Corps of Engrs., U. S. A., Bonneville, Ore.

[12a] Received by the Secretary July 25, 1939.

[13] "On the Behavior of Siphons," by J. C. Stevens, M. Am. Soc. C. E., *Transactions*, Vol. 99 (1934), . 986.

TABLE 3.—COMPARISON BETWEEN VELOCITIES COMPUTED BY THE AUTHOR'S FORMULA (EQUATION (11)) AND ACTUAL VELOCITIES

Item	Description	SIPHON NUMBER*			Average
		5	6	7	
	(*a*) DATA ON SELECTED SIPHONS*				
1	Discharge Q, in cubic feet per second	470.0	250.0	119.0	....
2	Operating head H, in feet	26.4	27.4	28.4	....
	Dimensions of Throat Section, in Feet:				
3	Width, b	6.03	4.52	2.68	....
4	Height, d	2.75	1.77	1.26	....
	Radius at the:				
5	Crest, r_c	2.12†	2.12†	2.12†	....
6	Mid-section, r_{av}	3.50	3.00	2.75	....
7	Summit, r_s	4.88†	3.88†	3.38†	....
	(*b*) VELOCITIES COMPUTED BY THE ROCK FORMULA (EQUATION (11))				
8	$\log_e r_s/r_c$	0.833	0.605	0.466	....
9	Discharge per unit width ($Q_1 = Q/b$), in cubic feet per second	78.0	55.3	44.4	....
	Computed Velocities in the Throat Section, in feet per second:				
10	Crest, V_c	44.1	43.2	45.0	44.1
11	Mid-section, V_{av}	26.7	30.6	34.6	30.6
12	Summit, V_s	19.2	23.6	28.2	23.7
	(*c*) ACTUAL VELOCITIES (SEE EQUATION (12))				
	Hydraulic Heads, in Feet:				
13	Elevation of energy line, B‡	764.8	765.0	760.6	....
14	H_c = Item 13 − 734.0§	30.8	31.0	26.6	....
	Elevation of points above the crest, at the:				
15	Crest, Z_c	0	0	0	....
16	Mid-section, Z_{av}	1.375	0.885	0.630	....
17	Summit, Z_s	2.75	1.77	1.26	....
	The ratio p/w,‖ at the:				
18	Crest	−28.8	−24.3	−30.9	....
19	Mid-section	−14.3	−14.2	−24.6	....
20	Summit	−10.7	−9.6	−17.0	....
	Velocity Heads,¶ at the:				
21	Crest	26.6	22.3	24.5	....
22	Mid-section	10.7	11.3	17.6	....
23	Summit	5.8	5.8	9.3	....
	Actual Throat Velocities, in Feet Per Second:				
24	Crest, V_c	41.4	37.9	39.7	40.0
25	Mid-section, V_{av}	26.2	27.0	33.7	29.0
26	Summit, V_s	19.3	19.3	24.5	21.0
	(*d*) COMPARISON OF ACTUAL VELOCITIES WITH THOSE COMPUTED BY EQUATION (11)				
	Percentage Differences by Equation (13):				
27	Crest = [(Item 10 − Item 24)/Item 24] 100	+6.5	+14.0	+13.4	+10.2
28	Mid-section = [(Item 11 − Item 25)/Item 25] 100	+1.9	+13.3	+2.7	+5.5
29	Summit = [(Item 12 − Item 26)/Item 26] 100	−0.5	+22.3	+15.1	+12.9

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* See *Transactions*, Am. Soc. C. E., Vol. 99 (1934), Fig. 3, p. 989.

† Assumed average values in order to start the radii at a common center.

‡ *Transactions*, Am. Soc. C. E., Vol. 99 (1934), Column (8), Table 3, p. 995.

§ Assume the datum at Elevation 734.0 to equal the elevation of the crest of all three spillways.

‖ *Transactions*, Am. Soc. C. E., Vol. 99 (1934), Columns (4), (7), and (10), Table 2, p. 994.

¶ Velocity head computed as the difference between the energy gradient and the hydraulic gradient.

used in Mr. Rock's formula to compute the velocities at the crest, mid-section, and summit of each siphon throat section. A second set of similar velocities at

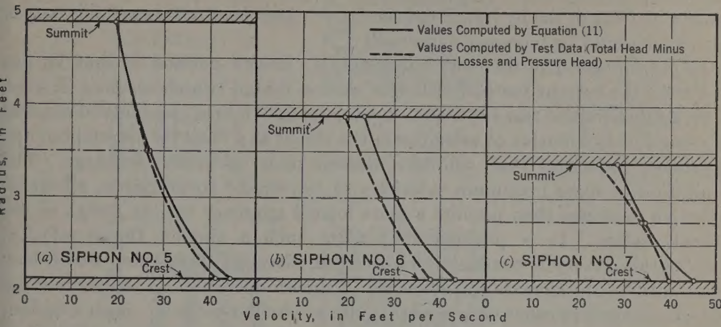

Fig. 4

these points was then computed by subtracting the actually measured hydraulic gradient from the energy gradient. That is,

$$\frac{V^2}{2\,g} = H_e - \left(Z + 33 + \frac{p}{w}\right) \qquad (12)$$

Comparison was finally made between the two sets of velocities by percentage difference in order to obtain an index as to the validity of Mr. Rock's formula.

Arithmetically:

$$\text{Percentage difference} = \frac{V_{\text{Rock}} - V_{\text{actual}}}{V_{\text{actual}}} \qquad (13)$$

It will be noted from Fig. 4 that in all three cases, the actual velocity values, as derived from the test data, are greater at the crest than at the summit, and that the velocity values as computed by the author's formula (Equation (11)) conform fairly well with the so-called actual values. When the number of ways in which experimental and personal error may have entered into the test data are considered, the comparison in Fig. 4 is believed to be surprisingly good; and, furthermore, this comparison appears to be a strong indication that the author's assumption of vortex flow is justified. In addition to the foregoing comparison, the practical application of Mr. Rock's formula was tested by checking the throat sections of several other existing siphon spillways. It was ound that the computed and actual values checked quite satisfactorily.

Further Considerations.—The author's treatment of the importance of proper lower leg design in high-head siphons is considered good. The mathematics involved in this phase, as well as those concerning the derivation of his vortex flow formula, were found to be valid. The numerical solution of the xample siphon, as given by Mr. Rock, aptly illustrates his point that a plot of he hydraulic gradient does not result in true negative pressures at the crest and summit of the throat section. However, if the author had included in his text the actual prototype measurements and model experiments, which he states prove conclusively that the negative pressures at the throat section as obtained from a plotting of the hydraulic gradient are erroneous, the value of this part of his paper would be increased materially.

Conclusions.—The inherent value of Mr. Rock's method is that he has extended the current method of throat section design (which assumes an even velocity distribution) an additional step forward. In brief, his method consists of using the distribution of velocities in the throat as a basis for determining the maximum velocities that will be obtained under a given discharge. The limitation of these maximum velocities to correspond to maximum allowable negative pressures then permits a more logical approach for the design of the throat section. It is possible that when such a siphon, throat-velocity, distribution theory is completely established, the next logical step would lead to an attempt to increase the efficiency of siphon operation by redesigning the throat section into other than a rectangular or circular section. Such a design, of course, would have to result in a gain in efficiency of operation sufficient to overcome increased construction costs.